Firm Heterogeneity and Adverse Selection in External Finance: Micro Evidence and Macro Implications*

Xing Guo
Bank of Canada
guoxing.econ@gmail.com

Pablo Ottonello
Maryland and NBER
ottonell@umd.edu

Thomas Winberry
Wharton and NBER
twinb@wharton.upenn.edu

Toni Whited
Michigan and NBER
toni.m.whited@gmail.com

July 3, 2025

Abstract

We study the macroeconomic consequences of asymmetric information between firms and external investors. To do so, we develop a heterogeneous firm macro model in which firms have private information about their quality. Private information creates a lemons problem in the market for external finance, depressing investment relative to the full information benchmark. We measure the distribution of private information, and therefore the magnitude of this lemons problem, using high-frequency stock price changes when firms raise new funding (revealing their quality to the market). We find that changes in distribution of private information are a quantitatively important determinant of aggregate fluctuations. For example, a spike in private information accounts for 40% of the decline in aggregate investment during the 2007-2009 financial crisis and made monetary stimulus significantly less effective at that time.

*We thank Andy Abel, Simon Gilchrist, Urban Jermann, Joao Gomes, Noemie Pinardon-Touati, Vladimir Smirnyagin, Phoebe Tian, Diego Zúñiga, and seminar participants at the Oxford Saïd-VU SBE Macro-finance Conference 2024, SED 2024, Chinese University of Hong Kong, University of Virginia, Penn-Wharton Macro Lunch, Stanford University, and Columbia University. We thank Jose Ignacio Cristi Le-Fort, Chaehyun Oh, Nabil Ryandiansyah, Mariana Sans, and Haoran Wang for excellent research assistance. Previously circulated as "Equity Financing and Monetary Policy." The views in this paper are those of the authors and do not reflect those of the Bank of Canada.

1 Introduction

Aggregate investment is one of the most volatile components of GDP, especially during financial crises. These fluctuations are concentrated among small firms which heavily rely on external finance to fund their investment. Together, these two observations have motivated the development of quantitative heterogeneous firm models to study the effect of financial shocks. To maintain tractability, the current generation of these models assumes that all sources of firm heterogeneity are fully observed by financial markets.

Our starting point is that, given the enormous degree of heterogeneity across firms in the data, firms likely have private information that is not observed by financial markets. Since Akerlof (1970), it is well understood that private information can lead to a lemons problem which reduces trade and, in extreme cases, causes the market to shut down entirely. Our main question is whether this mechanism, applied to the market for firms' external finance, is quantitatively important for understanding fluctuations in aggregate investment.

We answer this question by developing a heterogeneous firm model in which firms have private information, leading to a lemons problem in the market for external finance. A key challenge in quantifying the impact of the lemons problem is that, by its very nature, private information is not directly observable in the data. We overcome this challenge using the high-frequency change in a firm's stock price when it raises new financing. The idea behind our approach is simple: if stock prices reflect all available information, then the change in a firm's stock price reflects the revelation of any private information to the market.

Our main result is that *lemons shocks*, that is, changes in the distribution of private information over time, are an important determinant of aggregate investment fluctuations. Using our model, we infer the realized sequence of these lemons shocks from the observed sequence of high-frequency stock price changes in the data. We find that the dispersion of private information tripled during the 2007–2009 financial crisis and generates 40% of the total decline in investment seen in the data. Moreover, the rise in private information made firms less sensitive to changes in interest rates. Hence, lemons shocks are a particularly potent business cycle shock because they not only depress investment but also reduce the power of monetary stimulus to restore it.

We model private information as idiosyncratic shocks to the quality of the firm's installed capital, capturing changes in the value of capital that are not observable by the market.[1] This formulation of private information allows our model to match the fact that firms' stock prices typically fall when they raise financing, while other sources of private information, like news about future productivity, do not. We embed this formulation into a heterogeneous firm model in which firms accumulate capital following life-cycle dynamics and idiosyncratic productivity shocks. We close the model using a New Keynesian general equilibrium environment to study the effects of monetary policy.

The presence of private information creates a signaling game between firms, who observe their capital quality, and external investors, who do not. For our baseline analysis, we focus on equity financing because it is particularly information sensitive. Firms act in the interest of their existing shareholders, trading off the marginal benefit of issuing new shares—using the funds to invest in new capital—against the marginal cost—giving up a share of ownership in the existing capital stock. Under full information, firms and outside investors agree on the value of the existing capital, so the Modigliani-Miller theorem holds and firms undertake the frictionless level of investment. Private information distorts firms' incentives relative to this benchmark: low-quality firms would like to pool with high-quality ones to obtain cheaper financing, while high-quality firms would like to prevent this pooling from happening.

In equilibrium, high-quality firms signal their type by raising less equity than low-quality firms; this signal is costly because it leaves the high-quality firms with less funds to invest, but has the benefit of allowing their existing shareholders to retain a larger fraction of the capital stock. The resulting allocation is isomorphic to a model in which firms face an endogenous cost of issuing equity, which we call the *lemons wedge*. The shape of the lemons wedge does not resemble the exogenous equity issuance cost functions typically considered in the structural corporate finance literature, so the effects of private information are not well-approximated by those models.

[1] Private information is one of two classes of frictions studied in corporate finance theory (Tirole, 2006), the other being moral hazard among firm managers. The moral hazard friction has been widely used in macroeconomics as it underlies both the costly state verification model in, for example, Bernanke, Gertler and Gilchrist (1999) as well as the collateral constraint model in, for example, Kiyotaki and Moore (1997). We abstract from moral hazard to focus on developing a macroeconomic model in which the underlying financial friction stems from private information.

Because a firm's equity issuance decision fully reveals its capital quality to the market, the change in its stock price reflects the realization of its capital quality. Hence, the observed distribution of these price changes in the data reveals the unobserved distribution of capital quality among the subset of firms that issue equity. To extrapolate to all firms, we make the parametric assumption that capital quality is log-normally distributed.

We calibrate the distribution of capital quality using a panel of daily stock prices when firms announce a new seasoned equity offering. On average, a firm's stock price falls by 3.5% when it issues new equity, implying that issuing firms have substantially lower capital quality than average (reflecting the lemons problem). We choose the dispersion of capital quality to match this average price drop and then verify that untargeted features of the implied distribution are consistent with the data.

Quantitatively, the presence of private information lowers the steady-state capital stock by more than 5% compared to the full information benchmark. Most of this lost investment is concentrated among the small, highly-productive firms that face the largest lemons wedge. Hence, firm heterogeneity is critical to assessing the aggregate effects of private information. In fact, in both our model and the data, the net aggregate equity financing flow is negative, i.e., total payments to shareholders are larger than new equity raised. A representative firm model consistent with this fact would incorrectly conclude that the lemons wedge is zero.

We model lemons shocks as exogenous changes to the dispersion of capital quality across firms. In order to infer the realized time series of these shocks from the data, we linearize the model with respect to the aggregate states while still preserving a nonlinear approximation with respect to idiosyncratic states. The solution yields an observation equation mapping the underlying sequence of lemons shocks to the the time series of average stock price changes. We invert this mapping to recover the sequence of lemons shocks that exactly matches the time series of average price changes in the data. We then feed this inferred series of lemons shocks into the model and compare its predictions to the data. We particularly focus on the 2007–2009 Great Financial Crisis (GFC) because our procedure infers that the dispersion of capital quality more than tripled over this period.

Through the lens of our model, the observed lemons shocks during the GFC account for 40% of the total decline in investment in the data. Most of this decline is concentrated among the small which rely on external financing, as in the data. In fact, the lemons shock induces many firms to not raise equity at all, so they become less sensitive to changes in interest rates. For example, a 25 basis point rate cut during financial crisis would only generate 60% as much aggregate investment as it would have during the mid-1990s boom.

Finally, we extend our analysis to incorporate debt as another source of external finance in addition to equity. To provide a lower bound on the effect of private information in this environment, we assume that debt is completely information-insensitive in the sense that it is non-defaultable and is issued after the firm has raised equity (and has therefore revealed its capital quality to the market). We recalibrate the model to ensure that we continue to match average equity flows, as in our baseline, but also match average leverage. The lemons problem continues to be quantitatively important because our recalibration ensures that the same share of firms use equity as their marginal source of investment finance.

Related Literature Our findings contribute to our understanding of the aggregate effects of asymmetric information, which is also studied in, for example, Eisfeldt (2004), Kurlat (2013), Bigio (2015), and Bierdel et al. (2023). We make two main contributions to this literature. First, we incorporate firm heterogeneity, which is critical given that the lemons problem primarily affects small, highly productive firms. Second, we use high-frequency stock price changes to infer the degree of private information from the microdata.

Our paper also contributes to the corporate finance literature that studies the effects of financial frictions related to external finance. The most prominent strand of this literature assumes exogenous costs to issuing equity or debt (e.g. Gomes, 2001; Hennessy and Whited, 2007). In our model, private information creates an endogenous cost to issuing equity—the lemons wedge—which varies with firm-level and aggregate conditions. We find that the shape of this wedge is not well-approximated by the typical functional forms for exogenous cost functions considered in the literature. Our focus on private information, and its formulation in terms of unobserved asset quality, builds on a classic literature in corporate finance theory (e.g. Leland and Pyle, 1977; Myers and Majluf, 1984; DeMarzo and Duffie, 1999). We embed the insights of this literature into a quantitative infinite-horizon setting, discipline it with microdata, and study its macroeconomic implications.

Finally, our paper is broadly related to the macroeconomic literature that studies the effects of financial shocks in heterogeneous firm models (e.g. Khan and Thomas, 2013; Buera and Moll, 2015; Arellano, Bai and Bocola, 2017). We extend these models to incorporate private information and find that it is a quantitatively important source of financial shocks.[2]

Road Map The paper is organized as follows. Section 2 describes the baseline model and defines the equilibrium. Section 3 characterizes an equilibrium, delivering the representation of private information in terms of the lemons wedge. Section 4 calibrates the model, paying special attention to how we discipline the lemons wedge using the distribution of high-frequency stock price changes in the data. Section 5 shows that the implied steady-state losses from private information are large. Section 6 infers the sequence of lemons shocks from the data and shows that they reduce aggregate investment and render monetary stimulus less effective. Section 7 shows that our main quantitative results are robust to extending the model to incorporate non-contingent debt. Finally, Section 8 concludes.

2 Model

We build a heterogeneous firm model in which private information leads to a lemons problem in the market for external finance.

2.1 Heterogeneous Firms with Private Information

Time is discrete and infinite.

Technology Each period, there is a unit mass of firms indexed by $j \in [0, 1]$. Each firm produces y_{jt} units of an undifferentiated good using the production technology $y_{jt} = a_{jt} k_{jt}^{\alpha} \ell_{jt}^{1-\alpha}$ where a_{jt} is idiosyncratic productivity, k_{jt} is the firm's pre-existing capital stock, ℓ_{jt} is labor, and α is the elasticity of output with respect to capital. Productivity follows the log-AR(1) process $\log a_{jt} = \rho \log a_{jt-1} + \varepsilon_{jt}$, where $\varepsilon_{jt} \sim N(0, \sigma_a^2)$.

[2] Jeenas and Lagos (2024) provide evidence that equity financing significantly responds to monetary policy shocks. They show that an important transmission mechanism in these responses is a Tobin's q channel: changes in monetary policy affect the resale-value component of stock prices which in turn influences firms' equity issuance and investment decisions. Their channel is complementary to the one we study in this paper.

After production, a capital quality shock $\eta_{jt} \geq 0$ is realized. This shock determines the value of the firm's undepreciated capital $\eta_{jt}(1-\delta)k_{jt}$ going forward; hence, capital quality shocks are isomorphic to depreciation shocks, both of which are common in the macroeconomics literature.[3] One interpretation of these shocks are shocks to a specific plant or piece of equipment that cannot be easily uninstalled; for example, they could capture things like a decline in demand for the product produced by a particular plant or the loss of a key input supplier to that plant.

Capital quality is drawn from a distribution with discrete support that has N different points η_i and associated probabilities $\mathbb{P}(\eta_i)$. Later, we will assume that this distribution varies over time according to the *lemons shock* θ_t, which follows an exogenous stochastic process; we will impose specific functional forms later in our quantitative analysis. We assume that the realization of capital quality is i.i.d. across firms and independently distributed over time.[4] We normalize mean capital quality to 1 in each period.

Firms invest $x_{jt} \geq 0$ units of the final good in order to accumulate capital according to $k_{jt+1} = \eta_{jt}(1-\delta)k_{jt} + x_{jt}$. Investment entails the adjustment cost $\psi(x_{jt})$, also in units of the final good, so that total investment expenditures are $x_{jt} + \psi(x_{jt})$. The adjustment cost function $\psi(\cdot)$ is twice continuously differentiable with $\psi(0) = 0$, $\psi'(x) > 0$, and $\psi''(x) > 0$.

Information Structure A firm's idiosyncratic productivity a_{jt} and capital stock k_{jt} are publicly observable, but the realization of capital quality η_{jt} is private information to the firm. This assumption gives rise to a version of the typical lemons problem because raising finance requires firms to effectively sell a share of the revenues generated by their capital. Firms with low-quality capital have an incentive to pool with firms with high-quality capital to receive a better "price" than they otherwise would under full information.

We will show that raising funds is a negative signal about the firm's private information, allowing our model to be consistent with the fact that firms' stock prices typically fall when they raise new funds. If we had instead assumed private information was about idiosyncratic productivity, then raising funds may be a positive signal and therefore lead to a counter-

[3] Abel and Panageas (2024) develop a model with privately-observed depreciation shocks to study optimal financing of government spending.

[4] If a firm's capital quality were persistent, then firms' decisions would reveals the innovation to capital quality, which would have similar implications to our formulation here.

factual increase in stock prices. Similarly, if capital quality also affected the transformation of investment x_{jt} into new capital, then raising funds may again be a positive signal and lead to a counterfactual price increase. In this sense, our choices of how to model private information were strongly guided by the data on stock price changes.

External Finance For our baseline analysis, we assume that all external finance takes the form of new equity. In particular, in order to raise e_{jt} units of the final good from outside investment funds, the firm must provide them with newly issued shares (described below). In addition, if the firm raises $e_{jt} > 0$, it must also pay the exogenous equity issuance cost $\varphi(e_{jt})$ which captures direct costs of issuing new equity like underwriting fees. We assume $\varphi(e_{jt}) = \varphi_0 + \varphi_1 e_{jt}$, where φ_0 is a fixed cost and φ_1 is a linear cost. These exogenous costs generate realistic selection into equity issuance, which is important for matching the distribution of stock price changes when firms issue equity.

We focus on equity because it is the most information sensitive component of external finance. In principle, a similar information problem would apply to debt as well; for example, if some share of capital is lost in default, then low-quality firms would find it cheaper to issue debt and doing so would be a negative signal about the firm. However, the quantitative magnitude of this force would be smaller for debt than for equity because it only applies to states in which the firm defaults, i.e. debt is less information sensitive than equity.[5] To provide a lower bound on the effects of private information, Section 7 extends the model to incorporate debt that is completely information-insensitive.

Timing of Events The timing of events in each period t is as follows:

(i) Aggregate shocks are realized.

(ii) Firms receive an i.i.d. exit shock such that with probability ξ the firm must exit. Exiting firms transfer their capital stock k_{jt} to the household and permanently exit the economy. Each exiting firm is replaced by a new entrant, which is endowed with an initial level of productivity a_0 and an initial stock of capital k_0 from the household.

[5] In addition, quantifying this mechanism would require jointly modeling the signaling properties of debt and equity and disciplining that margin using high-frequency changes in bond yields.

The role of this entry and exit process is to capture the incentives of firms to invest as part of their lifecycle dynamics.

(iii) Idiosyncratic productivity shocks ε_{jt} are realized and production takes place. Firms use their existing stock of capital k_{jt} and hire ℓ_{jt} units of labor at real wage w_t to produce y_{jt} units of output. Firms sell this output to retailer firms (described below) at price p_t. The role of idiosyncratic productivity shocks is to capture incentives to invest that are orthogonal to lifecycle dynamics.

(iv) Capital quality η_{jt} is realized and firms raise new equity from investment funds. Investment funds do not observe capital quality η_{jt}.

(v) Firms invest in capital x_{jt} and pay dividends d_{jt} per share.

Firm Optimization The choice of labor in step (iii) solves the simple static problem $\max_{\ell_{jt}} p_t a_{jt} k_{jt}^{\alpha} \ell_{jt}^{1-\alpha} - w_t \ell_{jt}$, generating the decision rule $\ell_{jt} = \ell_t(a, k) \equiv \left(\frac{(1-\alpha)p_t a}{w_t}\right)^{\frac{1}{\alpha}} k$. Plugging this decision rule back into the static objective function gives variable profits

$$\max_{\ell_{jt}} p_t a_{jt} k_{jt}^{\alpha} \ell_{jt}^{1-\alpha} - w_t \ell_{jt} = A_t(a_{jt}) k_{jt}, \tag{1}$$

where $A_t(a_{jt}) = \left((1-\alpha)^{\frac{1-\alpha}{\alpha}} - (1-\alpha)^{\frac{1}{\alpha}}\right) (p_t a_{jt})^{\frac{1}{\alpha}} w_t^{-\frac{1-\alpha}{\alpha}}$. Variable profits are linear in capital because the production function is constant returns to scale, yielding two desirable properties for our analysis. First, the marginal product of newly-invested capital does not depend on the quality of existing capital η_{jt}; with decreasing returns, lower capital quality would mechanically raise the marginal product of new investment. Second, the linearity in (1) allows us to obtain analytical results which help characterize the equilibrium in Section 3. For example, we solve for the equilibrium decision rules under full information in closed form, which may be useful for other applications in heterogeneous firm models.[6]

[6]In typical heterogeneous firm models, the optimal scale of the firm is pinned down by decreasing returns to scale in production. In our model, the optimal scale is instead pinned down by the capital adjustment costs $\psi(x_{jt})$. One interpretation of our formulation is that span of control factors limits the ability of managers to create new lines of business, embodied in the flow of investment, instead of limiting their ability to manage existing lines of business, as in the decreasing returns formulation. Because the adjustment costs only depend on the flow amount of investment x_{jt}, our model matches key features of firm dynamics in the data, such as the facts that small and young firms grow faster than average (see Appendix C).
All that said, we could alternative specify a decreasing returns to scale production function for our quantitative work at the expense of losing the analytical tractability in Section 3.

We formulate the dynamic choices in steps (iv) and (v) recursively. The firm's individual state variables are a, its idiosyncratic productivity; k, its capital stock at the beginning of the period; and η, its capital quality. We denote the aggregate state variable, which includes the aggregate shocks and the distribution of firms over their individual states, as $\mathbf{s}_t$.

Firms choose their equity issuance in step (iv) in order to maximize the value to the initial shares outstanding at the beginning of the period (when the equity issuance decision is made). Let $v^0(a, k, \eta; \mathbf{s}_t)$ denote the present value of dividends to the current shares outstanding. For notational convenience, we suppress the dependence of the value function and other equilibrium objects on the aggregate state using time subscripts, i.e. $v_t^0(a, k, \eta) = v^0(a, k, \eta; \mathbf{s}_t)$ etc. Using this notation, the value function solves the Bellman equation

$$v_t^0(a, k, \eta) = \max_{e \geq 0} \frac{1}{1 + s_t(e; a, k)} v_t(e; a, k, \eta), \tag{2}$$

where $v_t(e; a, k, \eta)$ is the continuation value conditional on receiving e units of outside resources and $s_t(e; a, k)$ is the new shares that the investment funds demand for those resources (per initial share outstanding). The share demand schedule only depends on the publicly observable idiosyncratic state (a, k) and the firm's equity issuance decision e because investment funds do not observe capital quality η.

The post-issuance continuation value in step (v) solves the Bellman equation

$$v_t(e; a, k, \eta) = \max_{x, d \geq 0} d + \mathbb{E}_t \left[\Lambda_{t,t+1} \left\{ \xi k' + (1 - \xi) v_{t+1}^0(a', k', \eta') \right\} \right] \text{ such that} \tag{3}$$

$$x + \psi(x) + d = A_t(a)k + e - \varphi(e), \tag{4}$$

$$k' = \eta(1 - \delta)k + x, \tag{5}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor used to discount output conditional on the realization of the aggregate states in period $t+1$. Equation (4) is the flow-of-funds constraint for the firm; its expenditures on investment $x + \psi(x)$ or dividend payments d must be funded by the firm's variable profits $A_t(a)k$ or new equity e net of the issuance costs $\varphi(e)$.[7] Equation (5) is the law of motion for capital.

[7] In our baseline model, firms can only save resources from one period to the next by investing them in capital. Our extended model with debt in Section 7 allows firms to save in financial assets.

2.2 Investment Funds

Investment funds are perfectly competitive, so the share demand schedule is determined by the breakeven condition

$$\frac{s_t(e;a,k)}{1+s_t(e;a,k)}\mathbb{E}_t\left[v_t(e;a,k,\eta)|\mathcal{B}_t(\eta;e,a,k)\right]=e. \tag{6}$$

This condition says that the amount the investment fund provides to the firm (on the right-hand side) is exactly compensated by the value of the new shares the fund receives (on the left-hand side). These shares $s_t(e;a,k)$ entitle the fund to receive the fraction $\frac{s_t(e;a,k)}{1+s_t(e;a,k)}$ of the firms' dividends going forward, which has present value $v_t(e;a,k,\eta)$.

Since investment funds do not observe capital quality η, they must form beliefs $\mathcal{B}_t(\eta;e,a,k)$ to compute the expected present value of dividends. These beliefs $\mathcal{B}_t(\eta;e,a,k)$ are the investment funds' subjective probability that the firm has drawn a particular realization of capital quality η. This probability is conditional on the publicly available information about firm, namely its publicly observable states (a,k) and the amount equity it is raising e.

We require that investors' beliefs $\mathcal{B}_t(\eta;e,a,k)$ satisfy two standard consistency conditions. First, for equity issuance decisions e that occur along the equilibrium path, $\mathcal{B}_t(\eta;e,a,k)$ is determined by Bayes' rule. Second, for equity issuance decisions e that are not on the equilibrium path, $\mathcal{B}_t(\eta;e,a,k)$ must satisfy the D1 criterion from Banks and Sobel (1987). As usual in signaling games with $N>2$ unobservable types, we will use this criterion to refine the set of Perfect Bayesian equilibria. In the $N=2$ case, the D_1 Criterion coincides with the Intuitive Criterion from Cho and Kreps (1987), which states that investment funds assign zero probability $\mathcal{B}_t(\eta;e,a,k)=0$ to realizations η for whom the equity issuance choice e is equilibrium dominated, i.e., for whom the equilibrium choice gives the firm a higher payoff than the off-equilibrium choice e under any possible beliefs.

2.3 Equity Market Equilibrium

We are now ready to define an equilibrium of the signaling game in the equity market.

***Definition* 1.** *For each aggregate state $\mathbf{s}_t$ and publicly observable idiosyncratic state (a,k), an **equity market equilibrium** is a set of firm value functions $v_t^0(a,k,\eta)$ and $v_t(e;a,k,\eta)$;*

firm decision rules $e_t(a,k,\eta)$ and $k_t'(a,k,\eta)$; investment funds' share demand schedule $s_t(e;a,k)$; and investment funds' beliefs $\mathcal{B}_t(\eta;e,a,k)$ such that

(i) Firms optimize: given the share demand schedule, the value functions and decision rules solve the Bellman equation (2)—(5).

(ii) Investment funds break even: given firms' decisions and investment funds' beliefs, the share demand schedule $s_t(e;a,k)$ satisfies the breakeven condition (6).

(iii) Belief consistency: given firms' decisions, beliefs $\mathcal{B}_t(\eta;e,a,k)$ satisfy Bayes' rule and the D1 Criterion.

We will characterize this equilibrium in Section 3. The equilibrium determines firms' capital accumulation policies $k_t'(a,k,\eta)$ which can then be used to construct a law of motion for the distribution of firms over their idiosyncratic states (a,k,η). Since the realization of capital quality η is i.i.d. across firms, it suffices to track the marginal distribution $\mu_t(a,k)$ which integrates out capital quality.

2.4 New Keynesian Block and General Equilibrium

We embed this equity market equilibrium into a New Keynesian general equilibrium environment, allowing us to study the effects of monetary policy shocks. However, our results about the aggregate effects of lemons shocks also hold in a flexible price environment.

Retailers and Final Good Producer The heterogeneous firms described above sell their output to a fixed mass of retailers indexed by $i \in [0,1]$. Each retailer uses y_{it} units of the heterogeneous firms' undifferentiated output to produce a differentiated variety $\widetilde{y}_{it}$ using the technology $\widetilde{y}_{it} = y_{it}$; therefore, their real marginal cost is simply the relative price of the heterogeneous firms' output, p_t. Retailers then sell their differentiated variety as monopolistic competitors choosing their price $\widetilde{p}_{it}$ subject to a Calvo friction.

A competitive final goods producer purchases these differentiated varieties to produce final output Y_t using the constant elasticity of substitution (CES) production function

$$Y_t = \left(\int \widetilde{y}_{it}^{\frac{\gamma-1}{\gamma}} \, di \right)^{\frac{\gamma}{\gamma-1}}$$

where γ is the elasticity of substitution between varieties. As usual, this production function yields a CES demand schedule faced by the retailers. The final good is the numeraire.

Appendix A shows that the retailers and final good producer can be aggregated into a New Keynesian Phillips Curve (NKPC), which when linearized takes the familiar form

$$\log \Pi_t = \kappa \log \frac{p_t}{p^*} + \beta \mathbb{E}_t[\log \Pi_{t+1}] \tag{7}$$

where Π_t is gross inflation, $p^* = \frac{\gamma-1}{\gamma}$ is the steady state relative price of the heterogeneous firms' output, and κ depends on model parameters (including the degree of price stickiness). As discussed in Ottonello and Winberry (2020), the NKPC links nominal aggregate demand to heterogeneous firms' decisions through the relative price of the heterogeneous firms' output p_t. If aggregate demand Y_t increases, then the final goods producer demands more of the retailers' varieties $\widetilde{y}_{it}$; due to nominal rigidities, the retailers cannot increase prices one-for-one, so demand for the heterogeneous firms' output also increases. This force then increases the relative price of their output p_t, which then generates inflation through the NKPC (7).

Monetary Authority There is a monetary authority who sets the nominal gross interest rate R_t^{nom} following the Taylor rule

$$\log R_t^{\text{nom}} = \log \frac{1}{\beta} + \varphi_\pi \log \Pi_t + \varepsilon_t^{\text{m}}, \tag{8}$$

where $\varepsilon_t^m \sim N(0, \sigma_m^2)$ is the monetary policy shock and φ_π is the weight on inflation.

Representative Household There is a representative household with preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \chi N_t\right)$$

where β is the discount factor and the parameter χ controls the disutility of labor supply. The household owns all firms and investment funds in the economy and supplies labor.

Equilibrium An equilibrium of this model is a set of value functions and decision rules for the heterogeneous firms; the share demand schedule and beliefs of investment funds; a distribution of heterogeneous firms over their publicly observable states $\mu_t(a, k)$; macro aggregates Y_t, C_t, I_t, and N_t; and prices p_t, w_t, $\Lambda_{t,t+1}$, R_t^{nom}, and Π_t such that: (i) the equity market equilibrium in Definition 1 holds; (ii) the evolution of the distribution $\mu_t(a,k)$ is consistent with firms' capital accumulation decisions $k_t'(a,k)$ and the entry/exit process; (iii) p_t and Π_t follow the NKPC (7), (iv) R_t^{nom} follows the Taylor rule (8), (v) households optimize, and (vi) all markets clear.

3 Characterizing Equity Market Equilibrium

We now characterize the equity market equilibrium. We abstract from the exogenous equity issuance costs $\varphi(e)$ to focus on the distortions created by private information. Specifically, we set the fixed cost $\varphi_0 = 0$ and take the limit in which the linear cost $\varphi_1 \to 0$; this limit allows us to uniquely pin down financing decisions even under full information (they becomes indeterminate at the point $\varphi_1 = 0$). The proofs in Appendix A—and our quantitative analysis in Section 4 onward—allow for the general case of non-zero issuance costs $\varphi(e)$.

3.1 Full Information Benchmark

We begin by analyzing the full information benchmark in which investment funds observe the firm's capital quality η. This benchmark will be a useful point of comparison for assessing the distortions created by private information going forward.

***Proposition* 1** (Full information benchmark)**.** *Under full information, there is a unique equity market equilibrium in which the following properties hold:*

(i) The firm's pre-issuance value function is additively separable between existing capital and new investment:

$$v_t^0(a,k,\eta) = Q_t^k(a,\eta)k + Q_t^x(a), \tag{9}$$

where $Q_t^k(a,\eta) = A_t(a) + \eta(1-\delta)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(\xi + (1-\xi)Q^k(a',\eta')\right)\right]$ *is the present value of marginal products and scrap value of existing capital and* $Q^x(a) = -x_t^*(a) - \psi(x_t^*(a)) + \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\xi + (1-\xi)Q^k(a',\eta')\right)\right]x_t^*(a) + (1-\xi)\mathbb{E}_t[\Lambda_{t,t+1}Q_{t+1}^x(a')]$ *is the present value of current and future investment. The post-issuance value equals the pre-issuance value plus the value of the new equity:*

$$v_t(e;a,k,\eta) = Q_t^k(a,\eta)k + Q_t^x(a) + e. \tag{10}$$

(ii) The investment policy function is

$$x_t^*(a) = \psi'^{-1}\left(\mathbb{E}_t\left[\Lambda_{t,t+1}(\xi + (1-\xi)Q_{t+1}^k(a',\eta'))\right] - 1\right) \tag{11}$$

and is independent of capital k and capital quality η.

(iii) If revenues exceed investment expenditures $A_t(a)k \geq x_t^*(a) + \psi(x_t^*(a))$, then the financing policies are $e_t^*(a,k) = 0$ and $d_t^*(a,k) = A_t(a)k - x_t^*(a) - \psi(x_t^*(a))$. Otherwise, the financing policies are $e_t^*(a,k) = x_t^*(a) + \psi(x_t^*(a)) - A_t(a)k$ and $d_t^*(a,k) = 0$.

Proof. All proofs are in Appendix A. ∎

Proposition 1 provides a closed-form solution for firms' decisions under full information. To see how we arrive at this solution, first note that we can write the firm's pre-issuance value function (2) as

$$v_t^0(a,k,\eta) = \max_{e\geq 0} \frac{1}{1+s_t(e;a,k)} v_t(e;a,k,\eta) = \max_{e\geq 0} v_t(e;a,k,\eta) - \frac{s_t(e;a,k)}{1+s_t(e;a,k)} v_t(e;a,k,\eta), \tag{12}$$

which is the total post-issuance value of the firm net of the fraction given to investment funds. Since investment funds fully observe capital quality, their fraction of the firm's value exactly compensates for the resources they provide, $\frac{s_t(e;a,k)}{1+s_t(e;a,k)} v_t(e;a,k,\eta) = e$. Plugging this into the pre-issuance value (12) gives

$$v_t^0(a,k,\eta) = \max_{e\geq 0} v_t(e;a,k,\eta) - e.$$

Hence, under full information, the relative price of raising equity is 1, which equals the resource cost of investment goods. As a result, the pre-issuance value function and investment policy function coincides with the "frictionless" allocation in which firms choose investment directly (without specifying the financial contracts). This result is an example of Modigliani-Miller: the frictionless investment policy could in principle be financed in many ways; with equity contracts, firms are forced to finance it using promises of future revenues, and these promises are credible under full information.

Because the firm's variable profits are linear in capital, the pre-issuance value function (9) is additively separable into the value of the firm's existing capital, $Q_t^k(a,\eta)k$, and the value of its current and future investment, $Q_t^x(a)$. Furthermore, the value of existing capital equals the total amount of capital, k, times its marginal Q, $Q_t^k(a,\eta)$, which cumulates the expected present value of the marginal product and scrap value of capital over time.

Given this additive separability, the optimal investment decision is independent of existing capital and solves the subproblem

$$\max_x -x - \psi(x) + \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\xi + (1-\xi)Q^k(a',\eta')\right)\right]x,$$

where $Q^k(a',\eta')$ is the marginal Q of the investment once it becomes existing capital in the next period. Since capital quality is independently distributed over time, expected future Q is independent of current capital quality η. As a result, optimal investment $x_t^*(a)$ is also independent of capital quality.

In the limit where the equity issuance cost $\varphi_1 \to 0$, the financing policy functions minimize the amount of outside equity need to finance investment $x_t^*(a)$ subject to the non-negativity constraints $e \geq 0$ and $d \geq 0$.[8]

3.2 Equilibrium With Private Information

Under full information, Modigliani-Miller holds because investment funds fully observe the value of new shares issued by the firms; under private information, outside investors do not observe the value of new shares issued by the firm, creating a signaling game. Firms are the first movers in this game because they decide how much equity e to raise taking the investment funds' share demand schedule $s_t(e;a,k)$ as given. Therefore, it is equivalent to allow firms to choose both equity e and new shares s subject to the investment funds' breakeven condition. Using the formulation of the objective from (12), their problem is:

$$\max_{e\geq 0, s\geq 0} v_t(e;a,k,\eta) - \frac{s}{1+s}v_t(e;a,k,\eta) \quad \text{s.t.} \quad \frac{s}{1+s}\,\mathbb{E}_t\left[v_t(e;a,k,\eta) \mid \mathcal{B}_t(\eta;e,a,k)\right] = e. \tag{13}$$

Because investors' beliefs $\mathcal{B}_t(\eta;e,a,k)$ may shift discontinuously with the firm's choice of e, the optimization problem in (13) cannot be solved using standard Lagrangian methods.

[8] If $\varphi_1 = 0$ exactly, then any combination of equity issuance and dividend payments that satisfy the flow-of funds constraint (4) and the constraints $e \geq 0, d \geq 0$ would be a solution to the firm's problem.

Special Case for the Main Text The post-issuance value functions $v_t(e; a, k, \eta)$ in (13) are, in general, not additively separable under private information. Appendix A characterizes the equilibrium in this general case, but the expressions are cumbersome, so for the main text we focus on a special case which we call *one-shot private information.* In particular, we assume that capital quality is observable for all $t' > t$. In this case, the firm's problem reverts to full information after the equity issuance decision in period t, recovering the additive separability of the post-issuance value functions $v_t(e; a, k, \eta)$.

***Lemma* 1** (Post-Issuance Decisions Under One-Shot Private Information)**.** *Consider the one-shot private information environment. Conditional on raising e units of equity, the firm's investment policy and post-issuance value functions satisfy the following properties.*

(i) If the full-information investment policy is feasible, i.e. $x_t^*(a) + \psi(x_t^*(a)) \leq A_t(a)k + e$, then the investment policy is $x_t(e; a, k) = x_t^*(a)$, the dividend policy is $d_t(e; a, k) = A_t(a)k + e - x_t^*(a) - \psi(x_t^*(a))$, and the post-issuance value function is $v_t(e; a, k, \eta) = Q_t^k(a, \eta)k + Q_t^x(a)$, as in the full information benchmark.

(ii) Otherwise, the investment policy solves

$$x_t(e; a, k) + \psi(x_t(e; a, k)) = A_t(a)k + e \tag{14}$$

and is strictly increasing in equity e. The post-issuance value function is

$$v_t(e; a, k, \eta) = \widetilde{Q}_t^k(a)\eta(1 - \delta)k + \widetilde{Q}_t^k(a)x_t(e; a, k) + \widetilde{Q}_t^x(a), \tag{15}$$

where $\widetilde{Q}_t^k(a) = \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\xi + (1 - \xi)Q_{t+1}^k(a', \eta')\right)\right]$ is expected present value of capital starting in period $t + 1$ and $\widetilde{Q}_t^x(a) = \mathbb{E}_t\left[\Lambda_{t,t+1}(1 - \xi)Q_{t+1}^x(a')\right]$ is the expected present value of all future investment decisions starting in period $t + 1$.

If the full information policy $x_t^*(a)$ is feasible, the firm chooses it and pays out any remaining resources as dividends. If that policy is not feasible, the firm instead invests as much as possible (14) and pays no dividends. This case is the relevant one for firms that issue equity. Since equity is the marginal source of finance, investment is strictly increasing in equity $\frac{\partial x_t(e;a,k)}{\partial e} = [1 + \psi'(x_t(e;a,k))]^{-1} > 0$. The post-issuance value (15) is still additively separable but the expressions for the marginal Qs are slightly different because the non-negativity constraint on dividends binds in period t.

Equilibrium Characterization Lemma 1 characterizes choices conditional on an amount of equity raised e; we now turn to determining the optimal choices of e. The continuation value (15) highlights how the key tradeoffs depend on capital quality: the benefit of raising equity—higher investment $x_t(e;a,k)$—is independent of capital quality, but the cost of raising equity—giving up a fraction of firm ownership—is increasing in capital quality through the value of existing capital $\widetilde{Q}_t^k(a)\eta(1-\delta)k$. Appendix A shows that this tradeoff generates a single-crossing property which, together with the D1 criterion, allows us to construct a separating equilibrium.[9] The following result characterizes this type of equilibrium.

***Proposition* 2** (Equilibrium Under Private Information)**.** *Consider the one-shot private information environment. For a given publicly observable state $(a, k; \mathbf{s}_t)$, the equity market equilibrium satisfies the following properties:*

(i) If the full-information investment policy is feasible out of current profits, i.e. $x_t^*(a) + \psi(x_t^*(a)) \leq A_t(a)k$, then equity $e_t(a, k, \eta) = 0$ for all capital quality η.

(ii) Otherwise, the equity market equilibrium cannot be pooling, i.e. there is no equity issuance $\widehat{e} > 0$ such that $e(a, k, \eta_i) = e(a, k, \eta_j) = \widehat{e}$ for any two capital quality types $i \neq j$. Instead, a separating equilibrium exists in which the following holds:

(a) The lowest capital quality type η_1 achieves the full-information benchmark: $e_t(a, k, \eta_1) = e_t^*(a, k)$.

(b) For all other capital quality types $i = 2, 3, \ldots, N$, equity issuance is strictly decreasing in capital quality $e_t(a, k, \eta_i) < e_t(a, k, \eta_{i-1})$, the investment policy is given by (14), and the post-issuance value function is given by (15). The equity issuance policies can be constructed recursively:

[9] In general, the term "single crossing" refers to the fact that different types of agents' indifference curves cross at most once. In this model, firms' indifference curves in (e, s)-space are increasing because higher levels of resources e increase the total value of the firm, making it willing to accept a larger share given to outsiders s. Since giving up shares is costlier for high-quality firms, their indifference curves are steeper, meaning they intersect with those of low-quality firms at most once.

- *For capital quality type i, define the level of equity issuance $\overline{e}$ which makes the next-lowest type indifferent between choosing their equilibrium level of issuance $e_t(a, k, \eta_{i-1})$ and mimicking type i's issuance choice:*

$$\frac{1}{1 + s_t(a, k, \eta_{i-1})} v_t(e_t(a, k, \eta_{i-1}); a, k, \eta_{i-1}) = \frac{1}{1 + \overline{s}} v_t(\overline{e}; a, k, \eta_{i-1}) \tag{16}$$

  where $\overline{s}$ solves $\frac{\overline{s}}{1+\overline{s}} v_t(\overline{e}; a, k, \eta_i) = \overline{e}$.

- *Set $e_t(a, k, \eta_i) = \overline{e}$ and proceed to $i + 1$.*

(c) Investment funds beliefs are

$$\mathcal{B}_t(\eta; e, a, k) = \begin{cases} \mathbb{1}\{\eta = \eta_1\} & \textit{if } e > e_t(a, k, \eta_2) \\ \mathbb{1}\{\eta = \eta_i\} & \textit{if } e_t(a, k, \eta_{i+1}) < e \leq e_t(a, k, \eta_i) \textit{ for } i \geq 2 \\ 0 & \textit{else} \end{cases} \tag{17}$$

with the convention that $e_t(a, k, \eta_{N+1}) = 0$.

Part (i) of Proposition 2 shows that private information is irrelevant for the set of publicly observable states $(a, k, \mathbf{s}_t)$ in which firms can self-finance the full-information level of investment $x_t^*(a)$. Part (ii) characterizes equilibrium for the set of firms that cannot self-finance and, therefore, are affected by private information. These firms are the only ones which issue new equity.

The single-crossing property implies that there is no pooling equilibrium in which any two types of firms, call them η_{i_1} and $\eta_{i_2} > \eta_{i_1}$, issue the same amount of equity $e^{\text{pool}} > 0$. To see the intuition, define the threshold $\overline{e} < e^{\text{pool}}$ which makes the lower-quality firm indifferent between pooling and issuing $\overline{e}$, even if investment funds believed them to be the higher-quality firm at $\overline{e}$. Using the value function in (15), this threshold satisfies

$$\left(\frac{1}{1 + \overline{s}} - \frac{1}{1 + s^{\text{pool}}}\right) \eta_{i_1}(1 - \delta)k = \frac{1}{1 + s^{\text{pool}}} x_t(e^{\text{pool}}; a, k) - \frac{1}{1 + \overline{s}} x_t(\overline{e}; a, k), \tag{18}$$

where $\overline{s}$ denotes the number of shares investors demand if they believe the firm to be higher-quality, i.e. $\frac{\overline{s}}{1+\overline{s}} v_t(\overline{e}; a, k, \eta_{i_2}) = \overline{e}$. By construction, the benefit of retaining a larger share of its existing capital (the left-hand side of 18) exactly offsets the loss in new investment (the right-hand side 18) for the low-quality firm. High-quality firms have more valuable capital $\eta_{i_2} > \eta_{i_1}$, so the benefit strictly outweighs the cost. As a result, the high-quality firms are better off issuing equity e just below the threshold $\overline{e}$, while the low-quality firms are not.

This insight also determines the structure of a separating equilibrium. In such an equilibrium, firms' equity issuance decisions fully reveal their capital quality, so investment funds demand shares based on their true type. For this outcome to be consistent with investment funds' beliefs, each type must ensure that other types do not want to mimic them. In our equilibrium, this requirement implies the type-η_i firm chooses the level of equity issuance which leaves the next lowest type-η_{i-1} firm indifferent to mimicking them—exactly the threshold $\overline{e}$ which ruled out pooling in (18). The exception to this recursive structure is the lowest type η_1; because investment funds demand the most shares from this type, no other type wants to mimic them, and their problem is equivalent to the full-information benchmark.[10]

Since equity is the marginal source of investment finance for these firms, distortions in equity issuance $e_t(a,k,\eta)$ generate distortions in their investment $x_t(e_t(a,k,\eta);a,k)$ as well. We define the *firm-level losses from private information* as the gap between the full-information and actual investment policies $x_t^*(a,k) - x_t(e_t(a,k,\eta);a,k)$; these losses are zero for the lowest-quality firms η_1 and strictly increasing for higher levels of capital quality. Aggregating these losses across firms yields the *aggregate losses from private information* $\int \sum_i [x_t^*(a,k) - x_t(e_t(a,k,\eta);a,k)]\, \mathbb{P}(\eta_i) d\mu_t(a,k)$. We will quantify both the firm-level and aggregate losses from private information in our calibrated model.

Finally, equation (17) constructs beliefs for the investment funds which support this equilibrium and are consistent with the D1 criterion. If a type-η_i firm were to raise equity above its equilibrium level $e_t(a,k,\eta_i)$, then investment funds would believe them to be the next lower type η_{i-1} and therefore demand discontinuously more shares. These beliefs ensure that firms have the incentive to raise their equilibrium level of equity rather than mimicking another type. These incentives are summarized by the lemons wedges we derive below.

[10]Our recursive construction of the separating equilibrium follows the approach in Guerrieri, Shimer and Wright (2010). While we have not proved uniqueness of this equilibrium, it may be possible; this equilibrium is the natural one given the incentives described above and we have not been able to construct others.

General Case in Appendix Appendix A contains the general version of Proposition 2 in our full model with repeated private information and non-zero equity issuance costs $\varphi(e)$.[11] The general proposition is nearly identical to Proposition 2 with two exceptions. First, with repeated private information, the value functions are not additively separable. However, there still exist conditions under which the single-crossing property holds and therefore preserves the structure of this equilibrium. We numerically verify that these conditions are satisfied in our calibrated model.

Second, with non-zero equity issuance costs, it is possible that firms with different levels of capital quality will set $e = 0$ if the level of equity issuance implied by the no-mimicking condition (16) is too low to justify paying the fixed cost. In this case, there exists an *issuance cutoff* $\overline{\eta}_t(a, k)$ such that firms with $\eta \leq \overline{\eta}_t(a, k)$ issue positive equity while firms with $\eta > \overline{\eta}_t(a, k)$ do not. The issuance cutoff $\overline{\eta}_t(a, k)$ generates selection into equity issuance by capital quality, which is central to our calibration strategy.

3.3 Lemons Wedges

In our separating equilibrium, higher-quality firms are incentivized to restrict their equity issuance because investment funds would believe them to have lower quality if they were to issue more equity. We now show that this incentive is isomorphic to an endogenous equity issuance cost and find that the shape of this endogenous cost is not well-approximated by the exogenous cost functions typically studied in the literature.

Characterizing the Lemon Wedge To arrive at these endogenous costs, plug in the investment funds' breakeven condition to the firm's pre-issuance objective function (13):

$$v_t^0(a, k, \eta) = \max_{e \geq 0} v_t(e; a, k, \eta) - \frac{v_t(e; a, k, \eta)}{\mathbb{E}_t\left[v_t(e; a, k, \eta) \mid \mathcal{B}_t(\eta; e, a, k)\right]} \times e.$$

[11] We use this general version of the proposition to numerically solve the model. The fact that the firms' choices solve the no-mimicking condition (16) dramatically simplifies the solution relative to a model with defaultable debt. In those models, one must iterate between solving for firms' decisions given a debt-price schedule, then update the schedule using those decisions, and continue until convergence (which is not guaranteed). In our model, (the general version of) Proposition 2 determines firms' decisions and the share demand schedule simultaneously.

Hence, firms maximize their total post-issuance value net of the cost of raising new equity e; the relative price of equity is the true value of the firm existing owners give up relative to the value of the firms investment funds believe they are receiving. Defining $1 + \tau_t(e; a, k, \eta) = \frac{v_t(e;a,k,\eta)}{\mathbb{E}_t[v_t(e;a,k,\eta)|\mathcal{B}_t(\eta;e,a,k)]}$ establishes the following result:

Lemma 2 (Lemons Wedge)**.** *In an equity market equilibrium, the conditions for firm optimization and investment funds break even are summarized by*

$$\max_{e\geq 0} v_t(e; a, k, \eta) - (1 + \tau_t(e; a, k, \eta))e \text{ where } \tau_t(e; a, k, \eta) = \frac{v_t(e; a, k, \eta)}{\mathbb{E}_t \left[v_t(e; a, k, \eta) \mid \mathcal{B}_t(\eta; e, a, k)\right]} - 1. \quad (19)$$

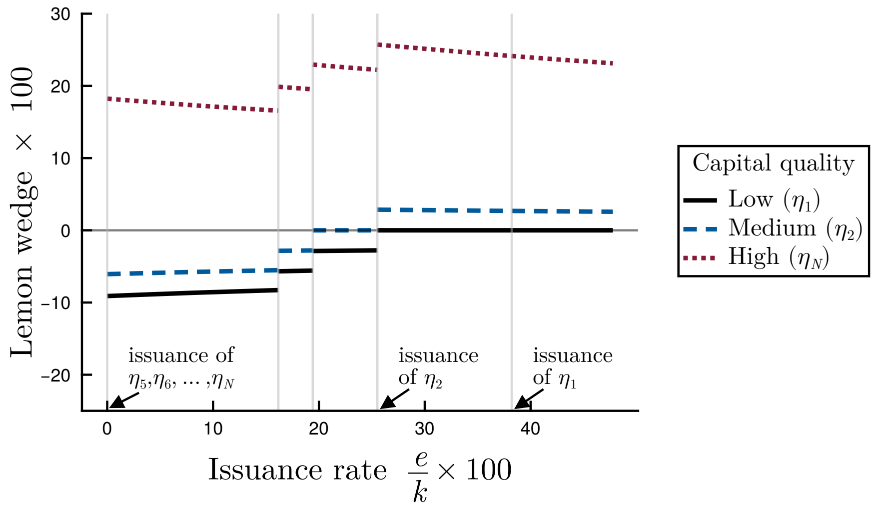
In the case with one-shot private information,

$$\tau_t(e; a, k, \eta) = \frac{\eta - \mathbb{E}_t \left[\eta \mid \mathcal{B}_t(\eta; e, a, k)\right]}{\mathbb{E}_t \left[\eta \mid \mathcal{B}_t(\eta; e, a, k)\right] + \frac{\widetilde{Q}_t^k(a)x_t(e;a,k)+\widetilde{Q}_t^x(a)}{\widetilde{Q}_t^k(a)(1-\delta)k}}. \quad (20)$$

We call $\tau_t(e; a, k, \eta)$ the *lemons wedge* because it summarizes how the cost of raising equity is distorted by private information. If a firm's true capital quality is higher than the investment funds believe, then $v_t(e; a, k, \eta) > \mathbb{E}_t \left[v_t(e; a, k, \eta) \mid \mathcal{B}_t(\eta; e, a, k)\right]$ and the lemons wedge is positive $\tau_t(e; a, k, \eta) > 0$. In this case, the firm is being "taxed" by private information because it has to give up more shares than it would under full information. Conversely, if the true capital quality is lower than the investment funds believe, then the firm is being "subsidized" because it does not have to give up as many shares as under full information.

In the special case with one-shot private information, the lemons wedge simplifies to the expression (20), which has two components. First, the lemons wedge is increasing in the gap between true and perceived quality, $\eta - \mathbb{E}_t \left[\eta \mid \mathcal{B}_t(\eta; e, a, k)\right]$. All else equal, a lower perceived quality increases the amount of new shares investment funds demand to provide a given amount of equity e, increasing the cost of raising that equity. Second, the lemons wedge is decreasing in the ratio of the value of current and future investment, $\widetilde{Q}_t^k(a)x_t(e; a, k) + \widetilde{Q}_t^x(a)$, to the value of existing capital, $\widetilde{Q}_t^k(a)(1 - \delta)k$. This property reflects the fact that private information concerns only the existing capital stock; if investment is a larger component of firm value, then private information about existing capital is less important.

FIGURE 1: Lemons Wedges



Notes: Steady state lemons wedges $\tau(e; a, k, \eta)$ as a function of equity issuance e for firms with the same publicly observable state but different levels of capital quality: low η_1, medium η_2, and high η_N from the discrete grid of capital quality. The y-axis is the lemons wedge expressed in percent $100 \times \tau(e; a, k, \eta)$ and the x-axis is the equity issuance rate expressed in percent $100 \times e/k$. Vertical bars denote equity issuance choices for different levels of capital quality η. Parameter values calibrated in Section 4.

Lemons Wedge vs. Exogenous Equity Issuance Costs Figure 1 plots the lemons wedge from the steady state of our calibrated model (though the general patterns hold for a range of parameter values). In particular, we plot $\tau_t(e; a, k, \eta)$ from equation (19) as a function of equity issuance e for firms with the same level of productivity a and capital k but different levels of capital quality η. In this figure, the vertical lines correspond to the equity issuance decisions $e(a, k, \eta)$ for different levels of capital quality η.

Consider a firm with the lowest level of capital quality, η_1, which issues the full-information level of equity. At this level, the lemons wedge is zero because firms are fairly priced at their equilibrium choices. However, the lemons wedge is non-zero for off-equilibrium choices; for example, if the firm were to issue equity at the level chosen by a firm with capital quality $\eta_2 > \eta_1$, then the lemons wedge would be approximately -4%, reflecting the fact that investors would incorrectly believe them to have higher capital quality. However, the firm does not make this choice because of the cost of foregone investment—illustrated on the right-hand side of (18)— is too large.

Firms with higher levels of capital quality do not issue the full-information level of equity because they would be effectively taxed by doing so. To understand why, consider a firm with the second-highest level of capital quality, η_2. If it were to issue at the full-information level, then investors would believe it to have lower capital quality $\eta_1 < \eta_2$ and therefore demand more new shares per unit of equity; the cost of giving up these additional shares is equivalent to a 4% tax. As a result, the firm reduces its equity issuance to its equilibrium point, where the lemons wedge discontinuously jumps down to zero.

A firm with the highest level of capital quality η_N would also be taxed by issuing the full-information level of equity, but the size of the tax is more than five times as large. Without the exogenous equity issuance costs $\varphi(e)$, this firm would issue a lower, but positive, amount of equity solving (16). However, with the issuance costs in our full model, the benefit from raising this low level of equity does not justify paying the fixed cost φ_0, so the firm does not issue equity. In other words, capital quality is above the issuance cutoff, $\eta_N > \overline{\eta}_t(a, k)$.

This discussion highlights two key features of the lemons wedge which are not well captured by the affine or quadratic equity issuance costs typically considered in the literature (see, e.g. Gomes, 2001; Hennessy and Whited, 2007). First, the lemons wedge varies with capital quality η while exogenous issuance costs do not. Second, the lemons wedge discontinuously jumps in order to induce firms to fully reveal their capital quality, while exogenous issuance costs are smoother. As a result, exogenous equity issuance costs cannot provide a good approximation of the effects of private information (despite the fact that private information is often cited as a rationale for these costs in the first place).

3.4 Inferring Private Information from the Data

Quantitatively, the distortions created by private information depend on the underlying distribution of capital quality. In order to calibrate this unobserved distribution, we relate it to the observed distribution of high-frequency stock price changes when firms issue equity. The logic of our approach relies on the fact that the equity market equilibrium is fully separating: if firms' equity issuance decisions reveal their capital quality to the market, and stock prices reflect the market's information about the firm, then the change in the stock price when a firm issues equity reflects the revelation of its capital quality.

We mimic the stock price change when firms issue equity by assuming that each period t has two subperiods, the morning and afternoon. The morning corresponds to steps (i) through (iii) in the timing of events from Section 2: all shocks are realized, entry/exit occurs, and production takes place, but equity issuance decisions have not yet been made. During this time, the firm's existing equity is traded at the morning price $p_t^m(a,k)$. In the afternoon, the firm makes its equity issuance decisions. If the firm raises positive equity $e_t(a,k,\eta) > 0$, it reveals its capital quality to the market and the afternoon price is equal to the amount of new equity per share, $p_t^a(a,k,\eta) = \frac{e_t(a,k,\eta)}{s_t(a,k,\eta)}$. If the firm does not raise new equity $e_t(a,k,\eta) = 0$, which may occur in our full model with fixed issuance costs $\varphi_0 > 0$, the afternoon price is the expected post-issuance value of the firm given investment funds' beliefs $p_t^a(a,k,\eta) = \mathbb{E}_t\left[v_t(0;a,k,\eta) \mid \mathcal{B}_t(\eta;0,a,k)\right]$. By no-arbitrage, the morning price is the expected value of the afternoon price, $p_t^m(a,k) = \sum_i \mathbb{P}(\eta_i)p_t^a(a,k,\eta_i)$.[12]

We define the *high-frequency price change* as the log-difference between the afternoon and morning price, $\Delta \log p_t(a,k,\eta) = \log p_t^a(a,k,\eta) - \log p_t^m(a,k)$, for the firms that issue positive equity $e_t(a,k,\eta) > 0$. Using the definition of the morning price,

$$\Delta \log p_t(a,k,\eta) = \log p_t^a(a,k,\eta) - \log \sum_{i=1}^{N} \mathbb{P}(\eta_i)p_t^a(a,k,\eta_i). \tag{21}$$

This high-frequency price change reflects the ex-post value of the firm given its realized capital quality relative to the ex-ante value of the firm averaging over the possible realizations of capital quality. Since the afternoon price is increasing in capital quality η, the distribution of these price changes is informative about the underlying distribution of capital quality.[13]

The high-frequency price change in (21) is more readily interpretable in the special case with one-shot private information and $N = 2$ levels of capital quality.

***Lemma* 3** (High-Frequency Price Change with One-Shot Private Information)**.** *Consider the one-shot private information environment with $N = 2$ capital quality types. The high-frequency price change is*

$$\Delta \log p_t(a,k,\eta) \approx \frac{\eta - \overline{\eta}}{\overline{\eta} + \frac{\widetilde{Q}_t^k(a)x_t(e_t(a,k,\eta_H);a,k) + \widetilde{Q}_t^x(a)}{\widetilde{Q}_t^k(a)(1-\delta)k}}, \tag{22}$$

[12]There is no discounting in this no-arbitrage condition because we assume the gap in time between the morning and afternoon is infinitesimal.

[13]If there were no private information, i.e. the dispersion of capital quality is zero such that $\eta = \overline{\eta}$ for all firms, then the firm's stock price will not change when it issues equity. Instead, the post-issuance value in (10) increases one-for-one with the amount of new equity raised, leaving the price per share unchanged.

where $\overline{\eta} = 1$ is the unconditional average of capital quality.

In this special case, the high-frequency price change (22) is proportional to the firm's realized capital quality η relative to the average capital quality $\overline{\eta} = 1$. Hence, the observed distribution of price changes is directly informative about the distribution of capital quality among the subset of firms that issue equity. Since only the firms with $\eta < \overline{\eta}_t(a, k)$ issue equity, these observations are negatively selected from the left tail of the capital quality distribution. Our calibration strategy, described in detail in Section 4, is to use the observed distribution of price changes to learn about the left tail of capital quality and then use functional form assumptions to extrapolate to the rest of the distribution.

4 Measuring Private Information

We calibrate the degree of private information to match the empirical distribution of high-frequency stock price changes. In this section, we focus on the steady state in which the distribution of capital quality is stationary; in Section 6, we allow the distribution to change over time in response to lemon shocks.

4.1 Empirical Distribution of High-Frequency Price Changes

Our measurement of the high-frequency stock price changes follows a classic literature in empirical corporate finance, such as Masulis and Korwar (1986) or Asquith and Mullins (1986). Our contribution is to use the distribution of these price changes to discipline the degree of private information in our model.

Data Sources We use data on seasoned equity offerings (SEOs) from the Securities Data Company (SDC) Platinum dataset.[14] These data contain information about the date when the firm first files the intention to pursue the SEO with the Securities and Exchange Commission (SEC), when the firm actually launches the issuance, and when the issuance is complete.

[14] We exclude initial public offerings from our sample because constructing the firm's stock price change requires observing a history of prices before the issuance.

We merge these data with the firm's daily stock price from the Center for Research in Securities Prices (CRSP). We clean the data in standard ways, described in Appendix B. Our final sample contains 3,178 SEOs between 1985q1 and 2018q4.

For each SEO in our sample, we define the *issuance event* as the day on which the firm first reveals it is raising new equity. The precise definition depends on whether the SEO is registered "on the shelf" or not. For non-shelf registered SEOs, which are the majority of observations before the mid-2000s, the firm's initial filing with the SEC commits it to issuing equity in a specific time frame. In these cases, we define the issuance event as the day of the initial filing. Second, for shelf-registered SEOs, which are the majority after the mid-2000s, firms do not need to commit to a specific timeline or amount of equity to raise (as long as it occurs within three years of filing). In these cases, we define the issuance event as the day the firm actually launches the issuance.

Price Changes The left panel of Figure 2 plots the average daily stock return around the equity issuance event. On average, the firm's stock price falls by 1.2% on the event day and another 2.3% on the next day; after that, the daily return reverts to zero, implying that the level of the stock price is permanently lower. The fact that most of the price change occurs the day after the issuance event reflects the fact that many filings occur after the end of the first trading day.[15] Overall, the issuance event is a clear outlier, consistent with a substantial amount of private information being released.[16]

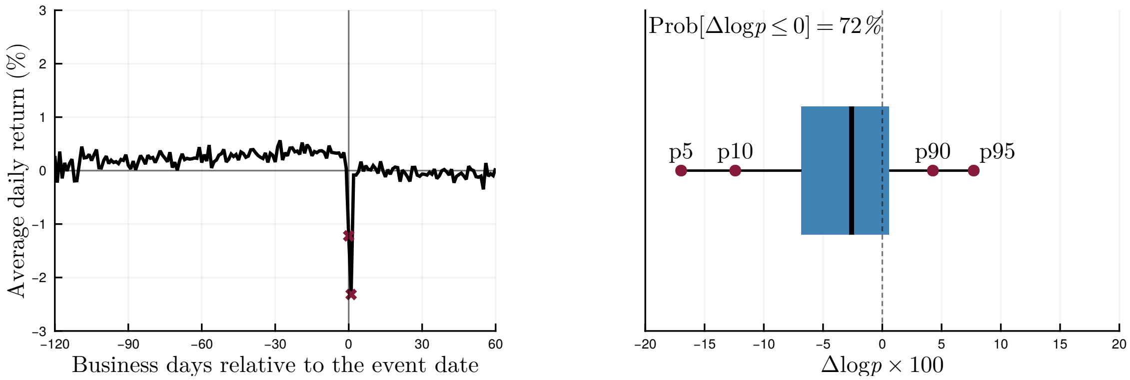
The right panel of Figure 2 presents a boxplot of the distribution of these high-frequency price changes across issuance events. We summarize the price change around a particular issuance event as the sum of the daily returns on the first and second day, which corresponds to the high-frequency log price change (21) from the model. Approximately 72% of issuance events see a negative price change, implying that the majority of issuing firms realize capital

[15]The average daily return before the issuance event is somewhat positive, suggesting that firms' stock prices tend to increase before they issue equity (as also found in the empirical corporate finance literature). As we discuss below, our model is consistent with this stock price runup because issuing firms are more likely to have received positive productivity shocks in the recent past.

[16]An alternative explanation for the price drop may be that firms face a downward-sloping demand curve for their equity, as in the "inelastic markets hypothesis" (Gabaix and Koijen, 2021). However, this hypothesis typically applies to demand for an entire asset class, not a specific firm's securities. In addition, Masulis and Korwar (1986) show that firms that eventually cancel their equity issuance—and therefore never change the supply of their securities in the market—still see a substantial price drop upon their initial announcement.

Figure 2: High-Frequency Stock Price Changes Around Issuance Events

(a) Average daily return | (b) Distribution of price changes



Notes: Panel (a) plots the average daily stock return of firms around an SEO issuance event defined in the main text. The red markers label the average daily return on the event day and the day after. Panel (b) presents a boxplot to visualize the distribution of the price changes across different issuance events. We define the price change as the sum of the daily return on the event day and the day after.

quality below its unconditional mean. The interquartile range runs from −6.8% to +0.6%, implying large dispersion in capital quality across firms.

Additional Analysis Appendix B contains additional analysis of these equity issuance events. Most importantly, we verify that firms actually use the new equity they raise for capital investment, as in our model. To do so, we merge the issuance events together with quarterly balance sheet data from Compustat and conduct an event-study analysis around the issuance event. We find that firms spend the majority of the equity they raise on capital investment, broadly consistent with our model; they use the remainder to either pay down debt or increase cash holdings, a pattern consistent with our extended model with debt. Appendix B also shows that our results about the price changes are robust to using other choices of the issuance event day and to using abnormal returns, which control for the firm's exposure to Fama and French (2015)'s five factor model, rather than the raw price change.

Calibration Strategy We choose the distribution of capital quality to match key features of the distribution of these high-frequency price changes from the data. In our specific implementation, we assume that capital quality follows a log-normal distribution $\eta \sim \log \mathcal{N}(-\frac{\sigma_\eta^2}{2}, \sigma_\eta^2)$. We discretize the grid of $\eta \in \{\eta_1, \ldots, \eta_N\}$ with $N = 10$ points running from $\eta_1 = e^{-3\sigma_\eta}$ to $\eta_N = e^{3\sigma_\eta}$. A key advantage of the log-normal family is that it is deter-

Table 1
Fixed Parameters

Parameter	Description	Value
Heterogeneous firms		
α	Capital share	0.36
δ	Capital depreciation rate	2.5%
ξ	Exit rate	2.1%
φ	Linear equity issuance cost	5%
New Keynesian block		
ψ_p	Probability do not change price	0.75
γ	Demand elasticity	10
φ_π	Monetary policy, response to inflation	1.5
β	Discount factor	0.99

Notes: parameters chosen to match external targets. A model period is one quarter.

mined by the single parameter σ_η. In Section 4.2, we choose σ_η to match the first moment of the price change distribution, i.e., the average price drop. In Section 4.3, we verify that higher-order features of the implied distribution of price changes resemble the data.

4.2 Parameterizing the Model to Match the Data

We calibrate the model in two steps. First, we fix some parameters to match standard macro targets. Second, we choose the remaining parameters so that the model's steady state matches key features of the micro data. A model period is one quarter.

Fixed Parameters Table 1 contains the parameters that we fix. The first set of parameters is related to the heterogeneous firm block of the model. We set the coefficient on capital in production $\alpha = 0.36$ and the capital depreciation rate to $\delta = 2.5\%$ quarterly. We set the exit probability $\xi = 0.021$ to match an annual exit rate of 8.4%, in line with the average firm exit rates in both Compustat and the Business Dynamics Statistics. Finally, we set the linear component of the equity issuance costs $\varphi_1 = 0.05$ to capture the directly measured issuance costs, such as underwriting fees, reported in Berk and DeMarzo (2007).

The second set of parameters in Table 1 are related to the New Keynesian block of the model. We set the probability that firms cannot adjust their price to 0.75, consistent with Nakamura and Steinsson (2008). We set the elasticity of substitution across varieties $\gamma = 10$

Table 2
Fitted Parameters

Parameter	Description	Value
Equity issuance frictions		
σ_η	Capital quality dispersion	0.06
φ_0	Fixed equity issuance costs	0.01
Adjustment Costs		
ψ_0	Scale parameter	1.30
ψ_1	Curvature parameter	0.78
Idiosyncratic productivity shocks		
ρ	Persistence	0.90
σ_a	SD of innovations	0.05
Firm lifecycle		
k_0	New entrants' capital stock	$0.22\ \times\mathbb{E}[k]$
a_0	New entrants' idiosyncratic productivity	$1.06\ \times\mathbb{E}[a]$

Notes: parameters chosen to match the targets in Table 3. The labor disutility parameter χ is calibrated to match the steady state level of employment at 1/3. A model period is one quarter.

to imply an average markup of approximately 11%. We set the coefficient on inflation in the Taylor rule (8) to $\varphi_\pi = 1.5$. Finally, we set the household's discount factor to $\beta = 0.99$ to generate a steady state annual real interest rate of 4%.

Fitted Parameters Table 2 contains the parameters that we choose to match moments in the data. As discussed above, the dispersion parameter σ_η determines the distribution of capital quality shocks. The fixed issuance cost φ_0 influences the selection into equity issuance. We parameterize the adjustment cost function to be isolelastic $\psi(x) = \frac{\psi_0}{1+\psi_1}x^{1+\psi_1}$, giving us the scale ψ_0 and curvature parameters ψ_1. The persistence ρ and standard deviation σ_a of the innovations to idiosyncratic productivity governs the importance of shocks to capital accumulation. Finally, the initial capital stock k_0 and idiosyncratic productivity a_0 of new entrants govern the importance of the firm lifecycle in capital accumulation.

We match moments about firm-level investment and financing behavior in our model to the corresponding moments in Compustat, a panel of publicly-traded firms.[17] At a conceptual level, our model of equity issuance applies to both private and public equity, not just the publicly-issued equity measured in Compustat. However, Compustat has two practical

[17] An alternative approach would be to explicitly model the selection of firms into Compustat and then compute the model-implied moments on this subsample.

advantages for our analysis: the data is readily available and it is relatively high quality because the accounts are subject to strict regulation, auditing, and reporting requirements. For that reason, the degree of private information among Compustat firms is likely much smaller than among other firms in the economy. Appendix B describes how we clean the Compustat data.[18]

Table 3 collects the moments that we target and shows the model matches them nearly exactly (despite the fact the model is overidentified due to nonlinearity). Although all moments jointly determine all parameters, we have found a strong association between certain moments and certain parameters.

Most importantly, the dispersion of capital quality σ_η is largely determined by the average price drop upon equity issuance. This moment is particularly informative, as equation (22) shows, it reflects the average capital quality among issuing firms compared to average capital quality in the population. As we showed in Section 3, only firms with capital in the left tail $\eta \leq \overline{\eta}(a, k)$ issue equity, so higher dispersion lowers the average capital quality among issuers.

The fixed issuance cost φ_0 is pinned down by the fact that, on average, firms issue equity less than once every five years. This target also helps ensure that we do not overstate the importance of equity financing for firms; though not targeted, the total amount of equity issuance per year turns out to be 1.8% of the capital stock compared 1% in the data.

The adjustment cost function $\psi(x)$ plays two roles in our analysis. First, it governs the sensitivity of investment with respect to changes in the incentive to invest; we discipline this role by matching the standard deviation of investment rates across firms. Second, it also governs the amount of dividends firms pay out to their shareholders because firms pay dividends whenever their current revenues exceed desired investment; we discipline this role by matching the aggregate dividend payout rate. Together, these targets help pin down the two adjustment cost parameters ψ_0 and ψ_1.

We discipline the process for idiosyncratic shocks using the firm-level cash flow rate $\frac{p^* y_{jt} - w^* \ell_{jt}}{k_{jt}}$, where p^* and w^* are the steady-state relative price of heterogeneous firms' output

[18] An important measurement issue is that the new equity issuance recorded in Compustat includes the exercise of stock options by employees; we eliminate this type of equity issuance flow by applying the filter proposed by McKeon (2015), ensuring our measurement only captures the equity issuance to external investors.

Table 3
Targeted Moments

Moment	Data	Model
Equity issuance frictions		
Average price drop	−3.5%	−3.4%
Frequency of equity issuance (annualized)	18%	18%
Investment frictions		
SD investment rate (annualized)	0.16	0.16
Dividend payout rate (annualized)	4.7%	4.8%
Idiosyncratic productivity shocks		
Autocorrelation of log cash flow rate (annualized)	0.70	0.69
SD of log cash flow rate (annualized)	0.38	0.37
Firm lifecycle		
Young vs. old log capital gap	−1.67	−1.67
Young vs. old growth rate gap	16.7pp	16.7pp

Notes: moments targeted to pin down the parameters in Table 2. The average price drop is the average price change described in the main text, weighted by new shares issued (in order to not over-emphasize small events). The frequency of equity issuance is the average of the fraction of firms with positive total equity issuance, after applying the filter proposed by McKeon (2015), in each year. The investment rate is computed as the ratio of capital expenditures (`CAPXQ`) to the lagged total book value assets (`ATQ`), expressed as an annual rate. The dividend payout rate is aggregated payout, which equals to the sum of cash dividends (`DVY`) and purchase of common and preferred stock (`PRSTKCY`), relative to lagged total book value assets, also expressed as an annual rate. The firm lifecycle targets the estimated $\hat{\beta}_{old}$ in regression (23), where y_{jt} is either log capital or the log capital growth rate.

and the real wage. Equation (1) implies that the steady-state log cash flow rate is equal to $\frac{1}{\alpha}\log a_{jt} + \log \widetilde{A}$, where $\widetilde{A}$ is constant. Hence, given a value of the capital coefficient α, the parameters of the AR(1) process for $\log a_{jt}$ are pinned down by the persistence and standard deviation of the firm-level cash flow rate.

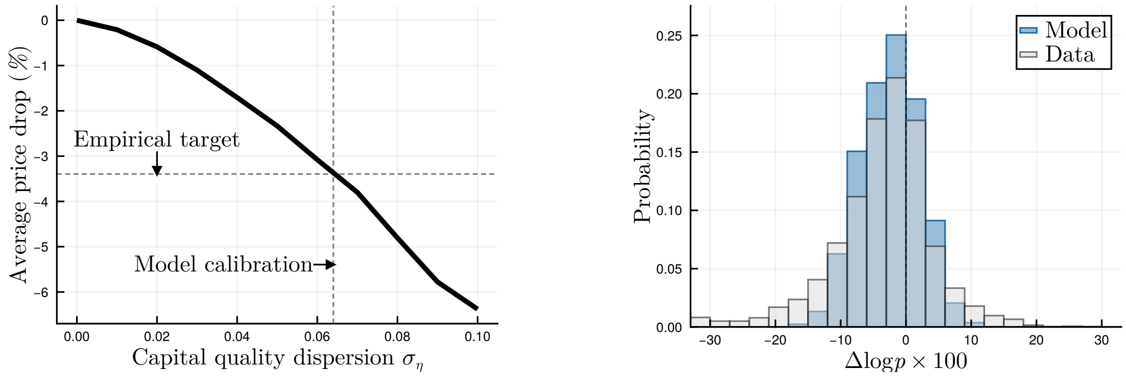
Finally, we discipline the lifecycle dynamics of firms using the regression

$$y_{jt} = \alpha + \gamma_t + \sum_{\iota=1}^{4} \beta_\iota \cdot \mathbf{1}[5 \times \iota < Age_{jt} \leq 5 \times (\iota+1)] + \beta_{old} \cdot \mathbf{1}[Age_{jt} > 25] + \epsilon_{jt}, \tag{23}$$

where y_{jt} is either log capital or the capital growth rate and γ_t is a time fixed effect. We target the coefficient β_{old}, which reflects the difference between the young (less than five years told) and old (greater than 25 years old) firms. The coefficient for log capital disciplines the initial capital stock of new entrants k_0. Given this value, the coefficient for the growth rate disciplines the initial productivity a_0 because higher productivity leads to faster growth.

Figure 3: Disciplining the distribution of capital quality shocks

(a) Identification Plot (b) Histogram of Price Changes



Notes: Panel (a) plots the steady state average price drop as a function of the dispersion of capital quality shocks σ_η, holding all other parameters fixed at their values in Tables 1 and 2. Panel (b) plots the histogram of price changes in the model (blue bars) and data (grey bars).

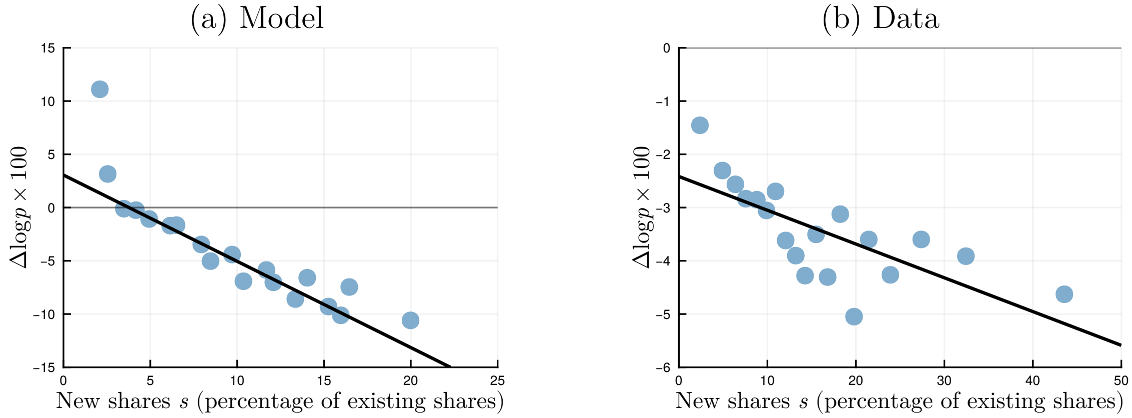
Table 2 contains the parameters that achieve this fit. The curvature of the adjustment costs is close to quadratic, which is a typical functional form. The persistence and standard deviation of productivity shocks are broadly in line with direct estimates from the productivity literature (see, e.g. Syverson, 2011). Appendix C analyzes the implied firm dynamics in our model. Small and young firms grow faster than average because they begin with a low initial capital stock k_0. Positive productivity shocks increase the marginal product of capital and induce similar growth dynamics. Both of these factors generate firm dynamics that are consistent with key features of the data.

4.3 Calibrated Distribution of Private Information

We conclude this section by validating our calibrated distribution of capital quality.

Matching the Average Price Drop We begin by showing that the average percentage price drop upon issuance locally identifies the dispersion parameter σ_η. To do so, the left panel of Figure 3 plots the model's average price drop against the value of the dispersion parameter σ_η, holding all other parameters fixed. The price drop is zero when there is no private information, $\sigma_\eta = 0$, and then grows monotonically as σ_η increases. Hence, the data's average price drop of -3.5% uniquely determines the model's parameter $\sigma_\eta = 0.06$.

Figure 4: Bin-scatter of price changes and new shares issued



Notes: Bin-scatters of the price change at each issuance event $100 \times \Delta \log p$ against the amount of new shares issued s, expressed as a percentage of the initial shares outstanding. Panel (a) plots the bin-scatter in the model together with the regression line with coefficient -0.81. Panel (b) plots the corresponding bin-scatter in the data together with the regression line with coefficient -0.06.

Validating Higher-Order Features of the Distribution The right panel of Figure 3 plots the histogram of price changes across equity issuance events in the model and the data. The model matches the overall shape of the empirical distribution, but its variance is smaller. While we could generalize the parametric family to exactly match both the mean and variance in the data, we choose to keep the log normal for the sake of parsimony. This choice is conservative because matching the total variance of price changes would require even more dispersed capital quality shocks, worsening the lemons problem.

Figure 4 plots bin-scatters of the high-frequency price changes against the amount of new shares issued. This correlation is at the heart of the single-crossing property from Section 3: issuing new shares is costlier for high-quality firms (who have a smaller price drop when they issue equity). Consistent with this mechanism, the left panel shows that the model generates a strong negative correlation between these two variables. The right panel shows they are negatively correlated in the data as well. Importantly, this result rules out formulations of private information in which issuing shares is a positive signal (which would imply a positive correlation) or models with pooling (which would imply no correlation).

Stock Price Runups Appendix B analyzes the dynamics of stock prices in the period before a firm issues equity in the data. As is well known, a firm's stock price tends to rise in the quarters before it issues equity, known as the "runup." A typical interpretation of this finding is that firms use market timing to mitigate the effects of private information; if stock prices are high for a nonfundamental reason, it is a good time to issue equity because the market will not penalize lemons as much.

Appendix C shows that our model also generates a stock price runup because of selection into equity issuance rather than market timing. In particular, firms are more likely to issue equity when their idiosyncratic productivity a is high. As a result, issuing firms have on average experienced a history of positive productivity shocks, generating the price runup.[19]

5 Steady State Losses from Private Information

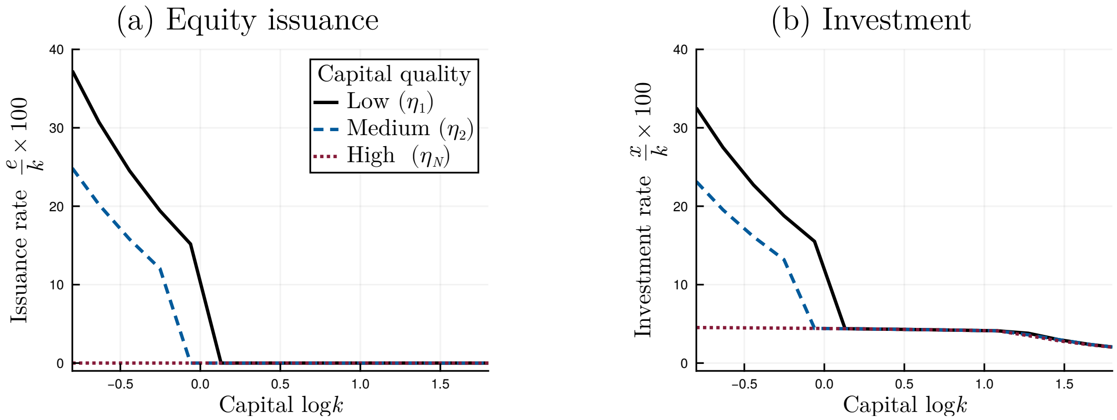
We now use our calibrated model to study the macroeconomic effects of private information. In this section, we show that the steady state losses from private information are large.

Firm-Level Losses From Private Information Figure 5 plots the equity issuance $e(a, k, \eta)$ and investment $x(a, k, \eta)$ decision rules as a function of capital k for firms with the same level of productivity a but different levels of capital quality η. Firms with the lowest level of capital quality η_1 provide a useful benchmark because, as Proposition 2 illustrates, their decision rules coincide with the full information allocation. For low levels of capital, the desired investment rate is high enough such that the firm must raise external equity to finance it. As the capital stock increases, the required equity issuance rate falls until the benefit of raising equity does not justify paying the fixed cost φ_0; at this point, the firm is constrained to finance its investment out of its revenues $A(a)k$. Finally, for sufficiently high levels of capital, the firm can completely finance its desired level of investment out of current revenues and pays out the difference as dividends.

Firms with higher levels of capital quality reduce their equity issuance to signal their type, as illustrated in Proposition 2. Consider the firms with the next lowest level of capital quality, η_2. These firms' equity issuance rates are depressed by up to one-half, and their investment rates by up to one-third, relative to the lowest-quality firms. In addition, the

[19] We also show that a larger price runup predicts a smaller price drop upon issuance in both the model and the data. This result occurs in the model because the issuance cutoff $\overline{\eta}(a, k)$ is increasing in idiosyncartic productivity a. As a result, average capital quality during issuance events following a larger runnup, leading to a smaller average price drop.

Figure 5: Firm-Level Effects of Private Information



Notes: Panel (a) plots the steady state equity issuance policy function $e(a, k, \eta)$ for a fixed value of idiosyncratic productivity a, as a function of capital k, for three different levels of capital quality η. The y-axis is the equity issuance rate expressed in percent, $100 \times e(a, k, \eta)/k$ and x-axis is the log capital stock $\log k$. Similarly, panel (b) plots the steady state investment policy function $x(a, k, \eta)$, expressed as the investment rate in percent, $100 \times x(a, k, \eta)/k$. The three different levels of capital quality are low η_1, medium η_2, and high η_N from the discrete grid of capital quality.

firms do not issue equity at all for a larger region of the state space. This general pattern continues as we consider higher levels of capital quality; for the highest level η_N, there is no level of capital such that they issue positive equity.

Of course, the firms that can self-finance the full-information level of investment are unaffected by private information. In other words, the lemons problem only impacts the firms that need to issue equity to finance investment, highlighting the importance of firm heterogeneity to our analysis. In fact, in the data, the net aggregate equity flow is negative, that is, firms pay out more dividends to their shareholders than they take in new equity. A representative firm model consistent with this fact would incorrectly imply that the lemons problem is irrelevant. Our model is also consistent with this fact, but the lemons problem still affects the smallest, most productive firms in the economy.

Aggregate Losses From Private information Table 4 quantifies the aggregate impact of private information by comparing the model's steady state equilibrium to the full information benchmark from Proposition 1. We find that these losses from private information are large: the steady state capital stock K^* is 5.6% lower than it would be under full information. The lower capital stock reduces labor demand, leading to both lower employment N^* (by 1.3%) and lower real wages w^* (by 1.5%). Together, the decline in capital and labor

TABLE 4
STEADY STATE LOSSES FROM PRIVATE INFORMATION

Capital stock	Employment	Wages	Output
-5.6%	-1.3%	-1.5%	-2.8%

Notes: steady state macro aggregates relative to full information benchmark from Proposition 1.

reduces aggregate output, Y^*, by 2.8%.

6 Business Cycle Effects of Lemons Shocks

We now study the aggregate effects of changes in the degree of private information over time.

6.1 Inferring Lemons Shocks from the Data

We infer lemons shocks from the observed average price drop series over time.

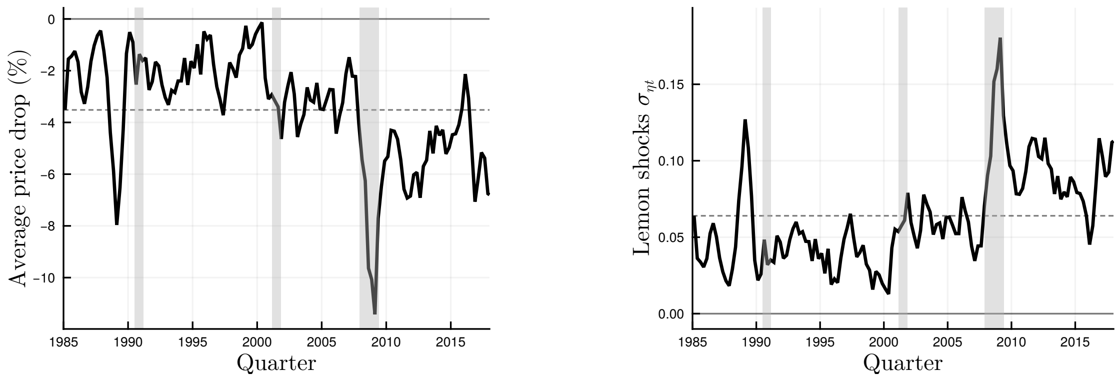
Average Price Drop Series The left panel of Figure 6 plots the time series of average price drops in our data. To compute this series, we first compute the average price drop among all issuance events within a quarter, and then compute a three-quarter moving average of the resulting series (to average out idiosyncratic variation due to small numbers of events in some quarters). The dashed line denotes the mean price drop -3.5% which identified the steady state dispersion of capital quality σ_η. However, there is a lot of variation around the mean, particularly during disruptions in financial markets like 1988 (the flash crash and savings and loan crisis), 1997 (the Asian financial crisis), 2001 (the dot-com bust), and 2007 (the Great Financial Crisis).

Modeling Lemon Shocks Given our log-normal functional form, we the lemons shock (θ_t from Section 2) is equal to the dispersion of capital quality, $\sigma_{\eta t}$. We assume that $\sigma_{\eta t}$ follows a log-AR(1) process around its steady state level σ_η:

$$\log \sigma_{\eta t} = (1 - \rho_\eta) \log \sigma_\eta + \rho_\eta \log \sigma_{\eta t-1} + \varepsilon_{\eta t}, \tag{24}$$

Figure 6: Time Series of Average Price Drops and Lemons Shocks

(a) Average price drop series (b) Inferred series of lemons shocks $\sigma_{\eta t}$



Notes: Panel (a) is the time series of the average price change among firms who issue equity in a given quarter, weighted by new shares issued (in order to not over-emphasize small events). We plot the three-quarter moving average to average out idiosyncratic variation due to small numbers of issuance events in some quarters. Panel (b) is the sequence of lemons shocks $\sigma_{\eta t}$ which we infer from equation (26) as described in the main text. Grey bars indicate NBER-dated recessions.

where $\varepsilon_{\eta t}$ is the innovation to the lemons shock process.[20] These innovations could capture changes in the actual risk to capital quality (for instance, if the demand or supplier relationships of an individual plant become riskier), or, more broadly, changes in investors' perceptions of those risks (in the spirit of Gorton and Metrick, 2012).

We solve for the aggregate dynamics generated by the lemons shocks using the Sequence Space Jacobian method developed by Auclert et al. (2021). This method computes a linear approximation around the aggregate state variables but preserves a fully global approximation with respect to individual state variables.[21] The method delivers an MA(T) representation of the model's dynamics, where T is chosen large enough to approximate the true MA(∞) process.

We use this linearized solution to construct an observation equation mapping the history of lemons shocks to the average price drop. To do so, let Δ_t denote the average price drop in period t relative to the average price drop in steady state. From our solution, the MA(T)

[20] To implement the lemons shock on our discrete grid, we hold the probabilities fixed and allow the location of the grid points to vary over time. In other words, we assume that lemons shocks actually affect how bad (or good) the set of possible outcomes are, not the probabilities of a fixed set of outcomes. Technically, we evenly stretch the distance between grid points while ensuring that mean capital quality is 1 and the standard deviation is $\sigma_{\eta t}$.

[21] Since the linear solution is certainty equivalent with respect to aggregate shocks, we do not need to specify the process for the innovations to lemons shocks $\varepsilon_{\eta t}$ in order to solve the model. However, we do need to impose a value for the persistence; we set $\rho_\eta = 0.87$, which turns out to match the persistence of the average price drop time series in our calibrated model.

representation for this variable is

$$\Delta_t = \sum_{h=0}^{T} \gamma_h \varepsilon_{\eta t-h}, \tag{25}$$

where $\varepsilon_{\eta t-h}$ are the history of innovations to the lemons shock process and γ_h are known from the linearized solution (in fact, they are the impulse response coefficients the average price drop to a one-time innovation to the lemons shock).

Inferring Lemon Shocks We invert the mapping (25) to recover the sequence of lemons shocks, which exactly match the data on average price drops. To do so, we stack the sequence of price drops over our sample into the vector $\boldsymbol{\Delta} = (\Delta_1, \dots, \Delta_S)'$, where $S = 136$ is the number of quarters in our sample. We similarly stack the sequence of innovations to the lemons shock into the vector $\mathbf{e}_\eta = (\varepsilon_{\eta 1}, \dots, \varepsilon_{\eta S})'$. We can then write the MA representation (25) for the observed sample in matrix form as $\boldsymbol{\Delta} = \Gamma \mathbf{e}_\eta$, where Γ is a $S \times S$ matrix comprised of γ_h coefficients. Since we have computed Γ as part of the linearization and can construct $\boldsymbol{\Delta}$ from our data, we can invert this equation to recover

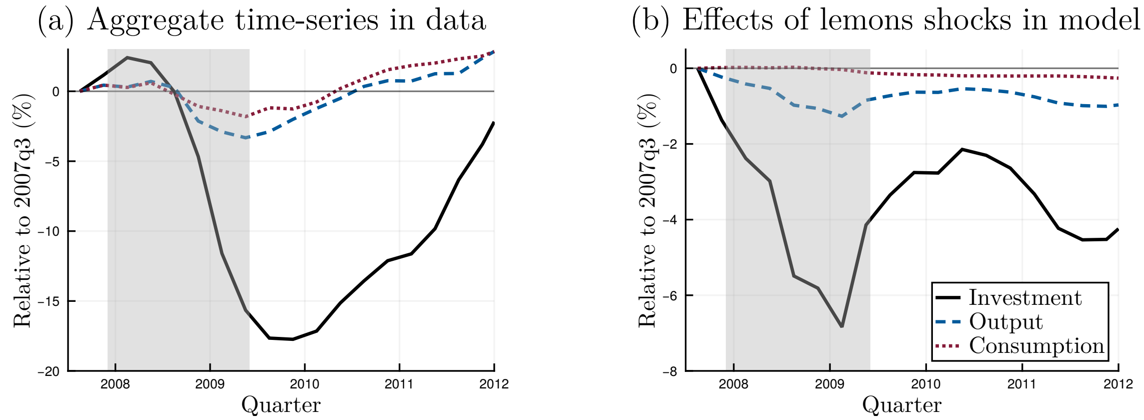
$$\mathbf{e}_\eta = \Gamma^{-1} \boldsymbol{\Delta}. \tag{26}$$

The right panel of Figure 6 plots the realized sequence of lemons shocks $\sigma_{\eta t}$ which we infer from equation (26). The dispersion of capital quality $\sigma_{\eta t}$ is higher during periods when the average price drop is larger. The reason is similar to the steady-state identification plot in Figure 3: higher dispersion $\sigma_{\eta t}$ implies that the average issuing firm has lower capital quality, leading to a larger price average price drop. Quantitatively, the time-series standard deviation of the lemons shock $\sigma_{\eta t}$ is 0.03, nearly half its average value $\sigma_\eta = 0.06$.

6.2 Aggregate Effects of Lemon Shocks During the GFC

We assess the aggregate effects of lemons shocks by feeding the realized sequence of shocks into our model and comparing the model's predictions for aggregate variables to the data. Appendix C shows that, over the entire sample, lemons shocks account for about a quarter

Figure 7: Aggregate quantities during the Great Financial Crisis



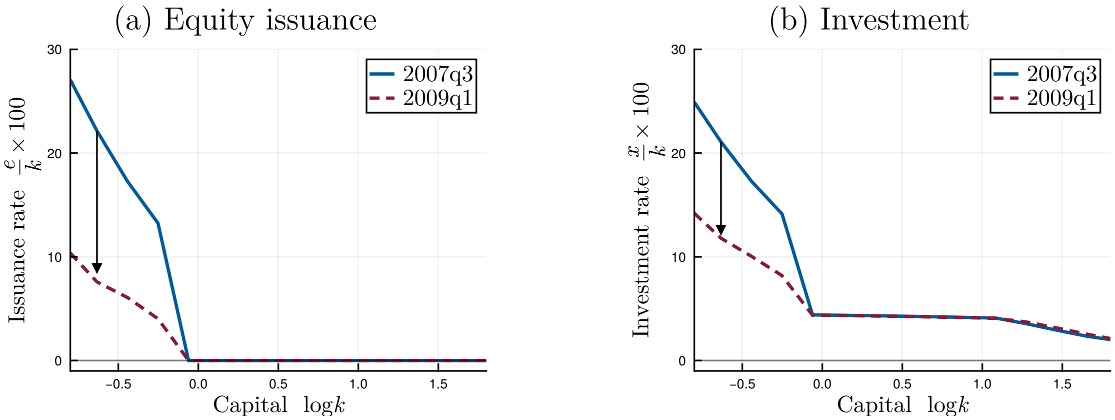
Notes: Panel (a) plots the time series of aggregate real non-residential fixed investment (FRED series `PNFI`), real GDP (FRED series `GDPC1`), and real personal consumption expenditures (FRED series `PCECC96`) between 2007q3 and 2012q1. Panel (b) plots the time series produced by our model in response to feeding in the realized sequence of lemons shocks $\sigma_{\eta t}$ we infer from (26) between 1985q1-2018q4, focusing on the 2007q3-2012q1 window. In both plots, the y-axis is the percentage deviation from 2007q3 and the grey bar is the NBER-dated recession.

of the unconditional volatility of investment seen in the data. However, Figure 6 suggests that the conditional volatility is likely higher during financial disruptions, when the lemons shock tends to be higher than average. Therefore, in this subsection, we focus on the Great Financial Crisis (GFC), which saw the largest spike in the lemons shock in our sample.

Aggregate Effects Figure 7 plots the time series of aggregate investment, GDP, and consumption during the GFC in the data and our model. In the data, investment fell approximately 18% peak-to-trough and took nearly four years to return to its pre-recession level. In the model, the lemons shocks generate a 7% decline in investment, accounting for 40% of the total decline in the data. We view this contribution as large given the myriad other shocks hitting the economy at the same time. Furthermore, the persistence of our inferred lemons shock series generates a persistent decline in investment, which is often difficult for other models of financial shocks to match (see the discussion in Khan and Thomas, 2013).[22]

[22]The decline in investment in our model generates a 2% decline in GDP, which is about one-third of the total decline from the data. However, the model predicts that consumption is relatively stable over this period. This stability is due to our assumption of sticky prices; with flexible prices, the decline in investment demand would lead to a counterfactual increase in consumption (see the related discussion in Justiniano, Primiceri and Tambalotti, 2010).

Figure 8: Effects of Lemon Shocks on Firms' Policies



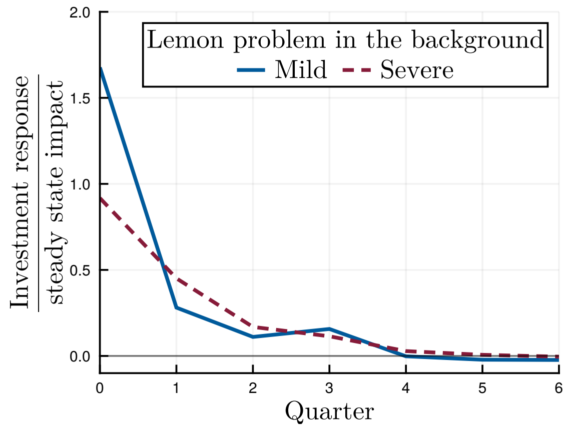
Notes: Policy functions at different points in path generated by feeding in the series of lemons shocks $\sigma_{\eta t}$ inferred from (26). Panel (a) is the equity issuance policy function $e_t(a, k, \eta_{2t})$ for fixed values of idiosyncratic productivity a and relative position of capital quality η_{2t} as a function of capital (see Footnote 20). The y-axis is the equity issuance rate expressed in percent, $100 \times e_t(a, k, \eta_{2t})/k$ and x-axis is the log capital stock $\log k$. Panel (b) is the investment policy function $x_t(a, k, \eta_{2t})$, expressed as the investment rate in percent, $100 \times x(a, k, \eta_{2t})/k$. In both panels, solid blue lines corresponds to $t = 2007q3$ and dashed red lines correspond to $t = 2009q1$.

Firm-Level Effects Figure 8 plots firm-level equity issuance and investment policies at the peak and trough of the GFC cycle as a function of firm size (holding fixed idiosyncratic productivity and its relative position in the capital quality grid). The lemons shock increases the gap between its capital quality and the next lowest level, tightening the no-mimicking constraint (16). As a result, firms must reduce their equity issuance and investment in the region where they were initially issuing equity. However, firms that were not issuing equity are largely unaffected by the lemons shock; hence, the decline in aggregate investment is driven by a decline among small firms, in line with the data.

6.3 Lemons Shocks Blunt Monetary Stimulus

We conclude this section by showing that lemons shocks dampen the effectiveness of monetary policy. We compute the impulse response of aggregate investment to a $\varepsilon_t^{\mathrm{m}} = -25$ basis point innovation to the Taylor rule which mean-reverts with quarterly autocorrelation 0.5. We compute these responses starting from three initial aggregate states: (i) steady state, (ii) an increase in the lemons shock $\sigma_{\eta t}$ which generates a larger average price drop comparable to the GFC, and (iii) a decrease in the lemons shock $\sigma_{\eta t}$ which generates a smaller average price drop comparable to the mid-1990s expansion. To allow for the impulse response to vary

Figure 9: State Dependence of Monetary Policy



Notes: Impulse response of aggregate investment to a $\varepsilon_t^{\mathrm{m}} = -25$ basis point innovation to the Taylor rule which then decays following an AR(1) process with autocorrelation 0.5. Solid blue line plots the impulse response if there is also negative lemons shock, which reduces the dispersion of capital quality to $\sigma_{\eta t} = 0.03$ (compared to the steady state value of $\sigma_\eta = 0.06$). Dashed red line is the impulse response if there is also a positive lemons shock which increases the dispersion of capital quality to $\sigma_{\eta t} = 0.13$. The y-axis reports these impulse responses relative to the impulse response to the monetary shock upon impact if there were no lemons shock (i.e. $\sigma_\eta = 0.06$ equals its steady state value).

across these states, we compute the fully nonlinear solution using an MIT shock approach.

Figure 9 shows that the monetary stimulus only generates 60% as much investment starting from the higher lemons shock $\sigma_{\eta t}$ relative to starting from the lower lemons shock. The reason is that the positive lemons shock induces more firms to not issue equity, making their investment insensitive to interest rates. Hence, our model provides a quantitative framework in which information problems make monetary policy less effective during financial crises.

7 Extended Model with Information-Insensitive Debt

We now show that our main results about the aggregate effects of private information from Sections 5 and 6 are robust to allowing firms to also raise non-contingent debt. Since we assume that debt is completely information insensitive, this extension provides a lower bound on the aggregate effects of private information.

Model Extension In our extended model, firms enter the period with outstanding debt b, which is public information. All firms repay their debt at the beginning of the period (before the realization of the exit shock). Firms can issue new debt b' after raising new equity, i.e.

during step (v) of the timing of events from Section 2.

This timing assumption abstracts from the signaling properties of debt and the associated complications described in Footnote 5 by making debt completely information-insensitive. Of course, if borrowing were frictionless, then the private information in the equity market would become irrelevant. We therefore assume that borrowing is limited by the collateral constraint $b' \leq \phi k'$, where ϕ governs the tightness of the constraint.[23]

The idiosyncratic state variable for the firm is now (a, k, b, η), where b is the amount of outstanding debt due at the beginning of the period. The firm's continuation value conditional on raising equity e now becomes

$$v_t(e; a, k, b, \eta) = \max_{x, d \geq 0, b'} d + \mathbb{E}_t \left[\Lambda_{t,t+1} \left\{ \xi (k' - \frac{b'}{\Pi_{t+1}}) + (1 - \xi) v^0_{t+1}(a', k', b', \eta') \right\} \right] \quad (27)$$

$$\text{s.t. } x + \psi(x) + d = A_t(a)k + e + \frac{b'}{1 + r_t(1 - \tau)} - b/\Pi_t, \quad (28)$$

$$k' = \eta(1 - \delta)k + x \text{ and } b' \leq \phi k' \quad (29)$$

where r_t is the risk-free real interest rate and the term $1 - \tau$ captures the tax advantage of debt.

The break-even condition for investment funds becomes

$$\frac{s_t(e; a, k, b)}{1 + s_t(e; a, k, b)} \mathbb{E}_t \left[v_t(e; a, k, b, \eta) | \mathcal{B}_t(\eta; e, a, k, b) \right] = e. \quad (30)$$

All else equal, the firm's continuation value is lower for firms with higher debt b, so investment funds require more new shares.

Recalibration Appendix D re-calibrates this extended model following the procedure in Section 4, with one additional parameter (the collateral constraint ϕ) and one additional target (the average leverage ratio across firms). Importantly, we match the same share of firms that issue equity each period as in the baseline model. However, the firms that issue equity tend to be levered, so they must issue more shares to raise the same amount of equity,

[23] We assume that $\phi \leq \eta_1(1 - \delta)$ to ensure that it is always feasible for firms to repay their outstanding debt at the beginning of the next period.

increasing the magnitude of the average price drop. As a result, the extended model requires lower capital quality dispersion $\sigma_\eta = 0.04$ to match the same average price drop.

Main Results Appendix D shows that this extended model with information-insensitive debt still implies large aggregate effects of private information, both in terms of the steady-state losses as well as the business cycle effects of the lemons shock. The reason is that the extended model has the usual "pecking order" of external finance: firms first prefer to finance investment using their own internal resources, then move onto debt, and only issue equity if their debt capacity is exhausted. As a result, equity is the marginal source of investment finance for any firm issuing equity; since the recalibration holds the share of firms issuing equity fixed, the same share of firms are affected by the lemons problem in the equity market. However, the magnitude of the lemons problem is somewhat smaller because our recalibrated dispersion of capital quality is lower than in the baseline model. Since the model abstracts from private information in debt issuance, these results represent a conservative lower bound on the macroeconomic effects of private information.

8 Conclusion

In this paper, we have argued that asymmetric information between firms and their external investors is an important determinant of aggregate investment. We made this argument in three main steps. First, we developed a heterogeneous firm business cycle model in which firms have private information, leading to a lemons problem in the market for external finance. Second, we developed a procedure to infer the quantitative magnitude of this lemons problem using the distribution of high-frequency stock price changes when firms raise new external financing. Third, we used a calibrated version of the model to show that empirically derived changes in the degree of private information account for a large share of business cycle fluctuations in aggregate investment.

We have kept our framework as simple as possible in order to highlight the novel contributions of our analysis. But given the enormous complexity of how firms operate in practice and the myriad ways in which these complexities vary across firms, other frictions likely interact with private information. An important one is moral hazard among firm managers, which is the other important class of frictions studied in corporate finance theory. This literature often argues that financing choices can constrain managers in a way that better aligns their incentives with shareholders. Our analysis suggests that these choices also carry important signaling properties and may interact with private information in important ways.

References

Abel, A. B., and S. Panageas (2024): "Optimal Financing of Government Purchases," *NBER Working Paper*, (w32961).

Akerlof, G. A. (1970): "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *The Quarterly Journal of Economics*, 84(3), 488–500.

Arellano, C., Y. Bai, and L. Bocola (2017): "Sovereign default risk and firm heterogeneity," Discussion paper, National Bureau of Economic Research.

Asquith, P., and D. W. Mullins, Jr. (1986): "Equity issues and offering dilution," *Journal of Financial Economics*, 15, 61–89.

Auclert, A., B. Bardóczy, M. Rognlie, and L. Straub (2021): "Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models," *Econometrica*, 89(5), 2375–2408.

Banks, J. S., and J. Sobel (1987): "Equilibrium Selection in Signaling Games," *Econometrica*, 55(3), 647–661.

Berk, J. B., and P. M. DeMarzo (2007): *Corporate finance*. Pearson Education.

Bernanke, B., M. Gertler, and S. Gilchrist (1999): "The Financial Accelerator in a Quantitative Business Cycle Framework," in *Handbook of Macroeconomics*, ed. by J. Taylor, and M. Woodford, vol. 1C, chap. 21, pp. 1341–1393. Elsevier.

Bierdel, A., A. Drenik, J. Herreño, and P. Ottonello (2023): "Illiquid Lemon Markets and the Macroeconomy," Discussion paper, National Bureau of Economic Research.

Bigio, S. (2015): "Endogenous liquidity and the business cycle," *American Economic Review*, 105(6), 1883–1927.

Buera, F. J., and B. Moll (2015): "Aggregate implications of a credit crunch: The importance of heterogeneity," *American Economic Journal: Macroeconomics*, 7(3), 1–42.

Cho, I.-K., and D. M. Kreps (1987): "Signaling games and stable equilibria," *The Quarterly Journal of Economics*, 102(2), 179–221.

DeMarzo, P., and D. Duffie (1999): "A liquidity-based model of security design," *Econometrica*, 67(1), 65–99.

Eisfeldt, A. L. (2004): "Endogenous liquidity in asset markets," *Journal of Finance*, 59(1), 1–30.

Fama, E. F., and K. R. French (2015): "A five-factor asset pricing model," *Journal of financial economics*, 116(1), 1–22.

Gabaix, X., and R. S. Koijen (2021): "In search of the origins of financial fluctuations: The inelastic markets hypothesis," Discussion paper, National Bureau of Economic Research.

Gomes, J. a. F. (2001): "Financing Investment," *American Economic Review*, 91(5), 1263–1285.

Gorton, G., and A. Metrick (2012): "Securitized banking and the run on repo," *Journal of Financial economics*, 104(3), 425–451.

Guerrieri, V., R. Shimer, and R. Wright (2010): "Adverse selection in competitive search equilibrium," *Econometrica*, 78(6), 1823–1862.

Hennessy, C. A., and T. M. Whited (2007): "How Costly Is External Financing? Evidence from a Structural Estimation," *Journal of Finance*, 62(4), 1705–1745.

Jeenas, P., and R. Lagos (2024): "Q-monetary transmission," *Journal of Political Economy*, 132(3), 971–1012.

Jorda, O. (2005): "Estimation and Inference of Impulse Responses by Local Projections," *American Economic Review*, 95(1), 161–182.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010): "Investment shocks and business cycles," *Journal of Monetary Economics*, 57(2), 132–145.

Khan, A., and J. K. Thomas (2013): "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity," *Journal of Political Economy*, 121(6), 1055–1107.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–248.

Kurlat, P. (2013): "Lemons markets and the transmission of aggregate shocks," *American Economic Review*, 103(4), 1463–89.

Leland, H. E., and D. H. Pyle (1977): "Informational asymmetries, financial structure, and financial intermediation," *The journal of Finance*, 32(2), 371–387.

Masulis, R. W., and A. N. Korwar (1986): "Seasoned equity offerings: An empirical investigation," *Journal of Financial Economics*, 15(1-2), 91–118.

McKeon, S. B. (2015): "Employee Option Exercise and Equity Issuance Motives," Working Paper.

Myers, S. C., and N. S. Majluf (1984): "Corporate financing and investment decisions when firms have information that investors do not have," *Journal of Financial Economics*, 13(2), 187–221.

Nakamura, E., and J. Steinsson (2008): "Five Facts about Prices: A Reevaluation of Menu Cost Models," *The Quarterly Journal of Economics*, 123(4), 1415–1464.

Ottonello, P., and T. Winberry (2020): "Financial heterogeneity and the investment channel of monetary policy," *Econometrica*, 88(6), 2473–2502.

Syverson, C. (2011): "What determines productivity?," *Journal of Economic literature*, 49(2), 326–365.

Tirole, J. (2006): *The Theory of Corporate Finance*. Princeton University Press, Princeton, NJ.

Online Appendix

A Theory Appendix

This appendix contains all proofs and additional theoretical discussion referenced in Section 3 of the main text. Section A.1 provides proofs of the general model with non-zero exogenous equity issuance costs $\varphi(e)$ and repeated private information. Section A.2 then provides additional results about our special case with no equity issuance costs and one-shot private information.

A.1 Full Model

We begin by outlining three key properties of the post-issuance value function $v_t(e; a, k, \eta)$ which serve as the foundation for our analysis. We then provide a detailed specification and proof of the separating equilibrium allocation. Finally, we prove that a pooling equilibrium does not exist. Throughout this section, we fix the publicly observable aggregate state $(a, k, \mathbf{s}_t)$; to simplify notation, we occasionally suppress dependence on the publicly observable state by denoting firm's policies by, e.g. $x_t(a, k, \eta_\iota) = x_\iota$, for $\iota = 1, 2, \ldots, N$.

A.1.1 Key properties of the post-issuance value function

There are three key properties of the value function $v_t(e; a, k, \eta)$ that play a crucial role in determining the equity market equilibrium:

(i) $\frac{\partial v_t(e;a,k,\eta)}{\partial e} > 0$ and $\frac{\partial v_t(e;a,k,\eta)}{\partial \eta} > 0$: The firm's value strictly increases with external equity funding and capital quality.

(ii) $\frac{\partial^2 v_t(e;a,k,\eta)}{\partial e^2} \leq 0 \ \forall e > 0$, and $\frac{\partial^2 v_t(e;a,k,\eta)}{\partial e^2} < 0 \ \forall e \in \left\{ e > 0 | \frac{\partial v_t(e;a,k,\eta)}{\partial e} > 1 \right\}$: The marginal return to external funding is non-increasing, and strictly decreasing when the marginal return of equity funding is above 1.

(iii) $\frac{\partial^2 v_t(e;a,k,\eta)}{\partial e \partial \eta} \leq 0$: Higher quality of existing capital does not make firms more effective in using the external equity funding to increase firm values.

In the special case from the main text, these three properties can be derived analytically. In the general case, we cannot analytically prove these properties, but we verify them numerically after solving the model.

The combination of properties (i) and (iii) lead to the single-crossing property:

***Lemma* 4** (Single-Crossing Property)**.** *If a firm with capital quality η weakly prefers issuance level e to $e' < e$, a firm with lower capital quality $\eta' < \eta$ will strictly prefer e to e'.*

Proof. Suppose a firm with capital quality η weakly prefers e to $e' < e$. Then, under a give share schedule $s_t(e; a, k)$, this implies:

$$\frac{1}{1 + s_t(e; a, k)} \cdot v_t(e; a, k, \eta) \geq \frac{1}{1 + s_t(e'; a, k)} \cdot v_t(e'; a, k, \eta). \tag{31}$$

Rewriting, we have:

$$\frac{v_t(e; a, k, \eta)}{v_t(e'; a, k, \eta)} \geq \frac{1 + s_t(e; a, k)}{1 + s_t(e'; a, k)}. \tag{32}$$

Define the function $f(\eta) \equiv \frac{v_t(e;a,k,\eta)}{v_t(e';a,k,\eta)}$. The sign of $f'(\eta)$ is given by:

$$\frac{\partial v_t(e; a, k, \eta)}{\partial \eta} \cdot v_t(e'; a, k, \eta) - v_t(e; a, k, \eta) \cdot \frac{\partial v_t(e'; a, k, \eta)}{\partial \eta}.$$

Using value function property (i) and (iii), we have:

$$\begin{aligned}
&\frac{\partial v_t(e; a, k, \eta)}{\partial \eta} \cdot v_t(e'; a, k, \eta) - v_t(e; a, k, \eta) \cdot \frac{\partial v_t(e'; a, k, \eta)}{\partial \eta} \\
&= v_t(e'; a, k, \eta) \cdot \left[\frac{\partial v_t(e; a, k, \eta)}{\partial \eta} - \frac{\partial v_t(e'; a, k, \eta)}{\partial \eta} \cdot \frac{v_t(e; a, k, \eta)}{v_t(e'; a, k, \eta)}\right] \\
&\leq v_t(e'; a, k, \eta) \cdot \left[\frac{\partial v_t(e'; a, k, \eta)}{\partial \eta} - \frac{\partial v_t(e'; a, k, \eta)}{\partial \eta} \cdot \frac{v_t(e; a, k, \eta)}{v_t(e'; a, k, \eta)}\right] \\
&= v_t(e'; a, k, \eta) \cdot \frac{\partial v_t(e'; a, k, \eta)}{\partial \eta} \cdot \left(1 - \frac{v_t(e; a, k, \eta)}{v_t(e'; a, k, \eta)}\right) \\
&< 0.
\end{aligned}$$

Therefore, $f'(\eta) < 0$, i.e., $f(\eta)$ is strictly decreasing in η. This implies:

$$\frac{v_t(e;a,k,\eta')}{v_t(e';a,k,\eta')} > \frac{v_t(e;a,k,\eta)}{v_t(e';a,k,\eta)} \geq \frac{1+s_t(e;a,k)}{1+s_t(e';a,k)}, \tag{33}$$

and thus:

$$\frac{1}{1+s_t(e;a,k)} \cdot v_t(e;a,k,\eta') > \frac{1}{1+s_t(e';a,k)} \cdot v_t(e';a,k,\eta'). \tag{34}$$

In other words, the firm with lower capital quality $\eta' < \eta$ strictly prefers higher issuance e to e'. ■

A.1.2 Characterization of Separating Equilibrium

We now state the characterization of the separating equilibrium, i.e. the general version of Proposition 2 in our full model.

***Proposition* 3** (Separating Equilibrium Under Private Information)**.** *For a given continuation value $v_t(e;a,k,\eta)$, define the one-shot full-information benchmark for a firm of type η_ι as the solution to:*

$$(e_\iota^*, s_\iota^*) \equiv \arg\max_{e\geq 0, s\geq 0} \frac{1}{1+s} \cdot v_t(e;a,k,\eta_\iota) \quad s.t. \quad \frac{s}{1+s} \cdot v_t(e;a,k,\eta_\iota) = e. \tag{35}$$

A separating equilibrium in the equity market can be characterized by the following recursive algorithm:

(i) Firms with the lowest capital quality η_1 issue equity as in the one-shot full-information benchmark:

$$e_1 = e_1^*, s_1 = e_1^*. \tag{36}$$

(ii) Suppose type-η_ι firms issue equity with $(e_\iota > 0, s_\iota > 0)$. Then the issuance of type-$\eta_{\iota+1}$ firms is bounded by the values $(\bar{e}_{\iota+1}, \bar{s}_{\iota+1})$ that satisfy $\bar{e}_{\iota+1} \leq e_\iota$ and:

$$\frac{1}{1+\bar{s}_{\iota+1}} \cdot v_t(\bar{e}_{\iota+1};a,k,\eta_\iota) = \frac{1}{1+s_\iota} \cdot v_t(e_\iota;a,k,\eta_\iota), \tag{37}$$

$$\frac{\bar{s}_{\iota+1}}{1+\bar{s}_{\iota+1}} \cdot v_t(\bar{e}_{\iota+1};a,k,\eta_{\iota+1}) = e_{\iota+1}. \tag{38}$$

These conditions ensure that type-η_ι firms are indifferent between mimicking the type-$\eta_{\iota+1}$ firms, and that investors break even under the belief that the firm is type-$\eta_{\iota+1}$.

The optimal issuance of type-$\eta_{\iota+1}$ firms is then:

$$e_{\iota+1} = \min\left\{e^*_{\iota+1}, \bar{e}_{\iota+1}\right\}, \tag{39}$$

$$s_{\iota+1} = \frac{e_{\iota+1}}{v_t(e_{\iota+1}; a, k, \eta_{\iota+1}) - e_{\iota+1}}, \tag{40}$$

provided that

$$\frac{1}{1+s_{\iota+1}} \cdot v_t(e_{\iota+1}; a, k, \eta_{\iota+1}) > v_t(0; a, k, \eta_{\iota+1}).$$

Otherwise, type-$\eta_{\iota+1}$ firms prefer not to issue equity, i.e., $e_{\iota+1} = 0$.

(iii) *If type-η_ι firms do not issue equity, then all firms with capital quality $\eta > \eta_\iota$ also choose not to issue equity.*

(iv) *The share schedule in the equilibrium is*

$$s_t(e; a, k) = \begin{cases} \frac{e}{v_t(e;a,k,\eta_1)-e} & \text{if } e > \hat{e}_2 \\ \frac{e}{v_t(e;a,k,\eta_\iota)-e} & \text{if } \hat{e}_{\iota+1} < e \le \hat{e}_\iota, \ \forall \iota \le \bar{\iota} \\ 0 & \text{if } e = 0 \end{cases} \tag{41}$$

and the associated investment fund's belief to support this equilibrium is

$$\mathcal{B}_t(\eta; e, a, k) = \begin{cases} \mathbb{1}\{\eta = \eta_1\} & \text{if } e > \hat{e}_2 \\ \mathbb{1}\{\eta = \eta_\iota\} & \text{if } \hat{e}_{\iota+1} < e \le \hat{e}_\iota, \ \forall \iota \le \bar{\iota} \\ \frac{\mathbb{P}(\eta_\iota)}{\sum_{\iota | e_t(a,k,\eta_\iota)=0} \mathbb{P}(\eta_\iota)} & \text{if } e = 0 \end{cases} \tag{42}$$

where $\bar{\iota} \equiv \max_\iota\{\iota | e_\iota > 0\}$ and the belief support cutoffs $\hat{e}_\iota$ is defined as:

- *$\hat{e}_\iota = \bar{e}_\iota$ if $e^*_\iota \ge \bar{e}_\iota$;*
- *$\hat{e}_\iota$ solves $\frac{v_t(\hat{e}_\iota;a,k,\eta_\iota)}{v_t(e_\iota;a,k,\eta_\iota)-e_\iota} = \frac{v_t(\hat{e}_\iota;a,k,\eta_{\iota-1})}{v_t(e_{\iota-1};a,k,\eta_{\iota-1})-e_{\iota-1}}$ if $e^*_\iota < \bar{e}_\iota$.*

In order to prove this proposition, we first prove four preparatory results that will be used in the proof of Proposition 3.

(i) Under the one-shot full information, the optimal equity issuance is decreasing in capital quality: that is, $e^*_t(a, k, \eta) \ge e^*_t(a, k, \eta')$ for any $\eta' > \eta$ if $e^*_t(a, k, \eta') > 0$.

Proof. Under the one-shot full information, firms solve the problem

$$e_t^*(a,k,\eta) \equiv \arg\max_{e \geq 0} v_t(e;a,k,\eta) - e. \tag{43}$$

If an interior optimum exists, it satisfies

$$s\frac{\partial v_t(e;a,k,\eta)}{\partial e}\bigg|_{e=e_t^*(a,k,\eta)} = 1. \tag{44}$$

Due to the property (ii) and (iii) of value function $v_t(e;a,k,\eta)$, it must be that $e_t^*(a,k,\eta) \geq e_t^*(a,k,\eta')$. Otherwise, if $e_t^*(a,k,\eta) < e_t^*(a,k,\eta')$, then

$$1 = \frac{\partial v_t(e;a,k,\eta')}{\partial e}\bigg|_{e=e_t^*(a,k,\eta')} < \frac{\partial v_t(e;a,k,\eta')}{\partial e}\bigg|_{e=e_t^*(a,k,\eta)} \tag{45}$$

$$\leq \frac{\partial v_t(e;a,k,\eta)}{\partial e}\bigg|_{e=e_t^*(a,k,\eta)}, \tag{46}$$

which contradicts the first-order condition for $e_t^*(a,k,\eta)$. ■

(ii) Under asymmetric information, the upper bound $\bar{e}_t(a,k,\eta')$ on equity issuance for type $\eta' > \eta$ imposed by the lemon threat from type η—as defined by equations (37) and (38)—exists and is strictly decreasing in η. That is, $\bar{e}_t(a,k,\eta') < \bar{e}_t(a,k,\eta)$ for any $\eta' > \eta$.

Proof. We first prove that $\bar{e}_t(a,k,\eta_{\iota+1}) < \bar{e}_t(a,k,\eta_\iota)$ for any $\iota = 1,2,\ldots,N-1$. The threshold $\bar{e}_t(a,k,\eta_{\iota+1})$ solves

$$v_t(\bar{e}_{\iota+1};a,k,\eta_\iota) - \bar{e}_{\iota+1} \cdot \frac{v_t(\bar{e}_{\iota+1};a,k,\eta_\iota)}{v_t(\bar{e}_{\iota+1};a,k,\eta_{\iota+1})} = v_t(e_\iota;a,k,\eta_\iota) - e_\iota. \tag{47}$$

Define the function

$$\phi(e) \equiv v_t(e;a,k,\eta_\iota) - e \cdot \frac{v_t(e;a,k,\eta_\iota)}{v_t(e;a,k,\eta_{\iota+1})} - \left[v_t(e_\iota;a,k,\eta_\iota) - e_\iota\right],$$

then we can prove that

- $\phi(e_\iota) > 0$ because
$$\phi(e_\iota) = e_\iota \cdot \left[1 - \frac{v_t(e_\iota;a,k,\eta_\iota)}{v_t(e_\iota;a,k,\eta_{\iota+1})}\right]$$
and the property (i) of function $v_t(e_\iota;a,k,\eta)$ implies $\frac{v_t(e_\iota;a,k,\eta_\iota)}{v_t(e;a,k,\eta_{\iota+1})} < 1$, $\phi(e_\iota) > 0$;
- and $\phi(0) < 0$ because
$$\phi(0) = v_t(0;a,k,\eta_\iota) - \left[v_t(e_\iota;a,k,\eta_\iota) - e_\iota\right] < 0,$$

otherwise type-η_ι would not chose to issue equity at the level of $e_\iota > 0$.

By continuity of $\phi(e)$, there exists an $\bar{e}_{\iota+1} \in (0, e_\iota)$ such that $\phi(\bar{e}_{\iota+1}) = 0$. Because $e_\iota \leq \bar{e}_\iota$, $\bar{e}_\iota < \bar{e}_\iota$ for any $\iota = 1, 2, \ldots, N-1$, which implies that $\bar{e}_t(a,k,\eta') < \bar{e}_t(a,k,\eta)$ for any $\eta' > \eta$. ■

(iii) Under asymmetric information, the value to existing shareholders

$$\begin{aligned} v_t^0(a,k,\eta) \equiv & \frac{1}{1 + s_t(e_t(a,k,\eta); a, k)} \cdot v_t(e_t(a,k,\eta); a, k, \eta) \\ = & v_t(e_t(a,k,\eta); a, k, \eta) - e_t(a,k,\eta) \end{aligned}$$

is strictly increasing in capital quality, i.e., $v_t^0(a,k,\eta) < v_t^0(a,k,\eta')$ for any $\eta' > \eta$.

Proof. From the non-mimicking condition, for any given ι,

$$v_t^0(a,k,\eta_\iota) = v_t(\bar{e}_{\iota+1}; a, k, \eta_\iota) - \bar{e}_{\iota+1} \cdot \frac{v_t(\bar{e}_{\iota+1}; a, k, \eta_\iota)}{v_t(\bar{e}_{\iota+1}; a, k, \eta_{\iota+1})} \tag{48}$$

$$= \frac{v_t(\bar{e}_{\iota+1}; a, k, \eta_\iota)}{v_t(\bar{e}_{\iota+1}; a, k, \eta_{\iota+1})} \cdot (v_t(\bar{e}_{\iota+1}; a, k, \eta_{\iota+1}) - \bar{e}_{\iota+1}) \tag{49}$$

$$< v_t(\bar{e}_{\iota+1}; a, k, \eta_{\iota+1}) - \bar{e}_{\iota+1} \tag{50}$$

$$\leq v_t(e_{\iota+1}; a, k, \eta_{\iota+1}) - e_{\iota+1} = v_t^0(a,k,\eta_{\iota+1}). \tag{51}$$

The first inequality follows from the property (i) of function $v_t(e; a, k, \eta)$, and the second from the definition of $e_t(a,k,\eta_\iota)$. ■

(iv) When $e_\iota^* < \bar{e}_\iota$, there exists a unique $\hat{e}_\iota \in (e_\iota^*, \bar{e}_\iota)$ that satisfies

$$\frac{v_t(\hat{e}; a, k, \eta_\iota)}{v_t^0(a,k,\eta_\iota)} = \frac{v_t(\hat{e}; a, k, \eta_{\iota-1})}{v_t^0(a,k,\eta_{\iota-1})}.$$

Proof. Define

$$f(e) \equiv \frac{v_t(e; a, k, \eta_\iota)}{v_t^0(a,k,\eta_\iota)} - \frac{v_t(e; a, k, \eta_{\iota-1})}{v_t^0(a,k,\eta_{\iota-1})}.$$

Then the derivative is

$$f'(e) = \frac{\partial v_t(e; a, k, \eta_\iota)/\partial e}{v_t^0(a,k,\eta_\iota)} - \frac{\partial v_t(e; a, k, \eta_{\iota-1})/\partial e}{v_t^0(a,k,\eta_{\iota-1})} < 0,$$

for two reasons: first, the property (iii) of function $v_t(e; a, k, \eta)$ implies that the marginal value of equity is weakly decreasing in η; second, $v_t^0(a,k,\eta_\iota) > v_t^0(a,k,\eta_{\iota-1})$

by the previous result. Since $f(e)$ is strictly decreasing, uniqueness and existence follow if we show $f(e_\iota^*) > 0$ and $f(\bar{e}_\iota) < 0$.

- We first show $f(\bar{e}_\iota) < 0$. To facilitate the proof, we rewrite $f(\bar{e}_\iota)$ as:
$$\begin{aligned} f(\bar{e}_\iota) = & \frac{v_t(\bar{e}_\iota; a, k, \eta_\iota)}{v_t^0(a, k, \eta_\iota)} - 1 - \frac{\bar{e}_\iota}{v_t(\bar{e}_\iota; a, k, \eta_\iota) - \bar{e}_\iota} \\ & - \left[\frac{v_t(\bar{e}_\iota; a, k, \eta_{\iota-1})}{v_t^0(a, k, \eta_{\iota-1})} - 1 - \frac{\bar{e}_\iota}{v_t(\bar{e}_\iota; a, k, \eta_\iota) - \bar{e}_\iota} \right]. \end{aligned}$$
The non-mimicking condition (37) implies
$$\begin{aligned} & \frac{v_t(\bar{e}_\iota; a, k, \eta_{\iota-1})}{v_t^0(a, k, \eta_{\iota-1})} - 1 - \frac{\bar{e}_\iota}{v_t(\bar{e}_\iota; a, k, \eta_\iota) - \bar{e}_\iota} \\ = & \frac{v_t(\bar{e}_\iota; a, k, \eta_{\iota-1})}{v_t(\bar{e}_\iota; a, k, \eta_{\iota-1}) - \bar{e}_\iota \frac{v_t(\bar{e}_\iota; a, k, \eta_{\iota-1})}{v_t(\bar{e}_\iota; a, k, \eta_\iota)}} - 1 - \frac{\bar{e}_\iota}{v_t(\bar{e}_\iota; a, k, \eta) - \bar{e}_\iota} \\ = & \frac{v_t(\bar{e}_\iota; a, k, \eta_\iota)}{v_t(\bar{e}_\iota; a, k, \eta_\iota) - \bar{e}_\iota} - 1 - \frac{\bar{e}_\iota}{v_t(\bar{e}_\iota; a, k, \eta_\iota) - \bar{e}_\iota} = 0. \end{aligned}$$
Meanwhile, the optimality of $e_t^*(a, k, \eta_\iota)$ implies
$$v_t^0(a, k, \eta_\iota) > v_t(\bar{e}_\iota; a, k, \eta_\iota) - \bar{e}_\iota$$
and
$$\frac{v_t(\bar{e}_\iota; a, k,_\iota)}{v_t^0(a, k,_\iota)} - 1 - \frac{\bar{e}_\iota}{v_t(\bar{e}_\iota; a, k,_\iota) - \bar{e}_\iota} = \frac{v_t(\bar{e}_\iota; a, k, \eta_\iota)}{v_t^0(a, k, \eta_\iota)} - \frac{v_t(\bar{e}_\iota; a, k, \eta_\iota)}{v_t(\bar{e}; a, k, \eta_\iota) - \bar{e}_\iota} < 0,$$
so $f(\bar{e}_\iota) < 0$.

- To show $f(e_\iota^*) > 0$, we must verify that
$$\frac{v_t(e_\iota^*; a, k, \eta_\iota)}{v_t(e_\iota^*; a, k, \eta_\iota) - e_\iota^*} > \frac{v_t(e_\iota^*; a, k, \eta_{\iota-1})}{v_t^0(a, k, \eta_{\iota-1})},$$
which is equivalent to
$$v_t^0(a, k, \eta_{\iota-1}) > v_t(e_\iota^*; a, k, \eta_{\iota-1}) - \frac{v_t(e_\iota^*; a, k, \eta_{\iota-1})}{v_t(e_\iota^*; a, k, \eta_\iota)} \cdot e_\iota^*.$$
The function on the right-hand side,
$$v_t(e; a, k, \eta_{\iota-1}) - \frac{v_t(e; a, k, \eta_{\iota-1})}{v_t(e; a, k, \eta_\iota)} \cdot e,$$
is increasing in e for $e \leq e_t^*(a, k, \eta_{\iota-1})$. Hence, the above inequality holds and $f(e_\iota^*) > 0$.

■

We now proceed with the proof of Proposition 3, which consists of three main steps:

(i) The equity issuance schedule $s_t(e;a,k)$ defined in (41) is consistent with the belief $\mathcal{B}_t(\eta;e,a,k)$ as specified in (42).

(ii) The issuance choices $\{e_t(a,k,\eta_\iota)\}_{\iota=1}^n$ are optimal for firms under the issuance schedule $s_t(e;a,k)$.

(iii) The belief $\mathcal{B}_t(\eta;e,a,k)$ is consistent with firms' equilibrium issuance behavior and satisfies the D1 criterion for off-equilibrium issuance choices.

Step (i): consistency between $s_t(e;a,k)$ and $\mathcal{B}_t(\eta;e,a,k)$ Given the belief function $\mathcal{B}_t(\eta;e,a,k)$, the conditional expectation of the firm's value is

$$\mathbb{E}[v_t(e;a,k,\eta) \mid \mathcal{B}_t(\eta;e,a,k)] = \begin{cases} v_t(e;a,k,\eta_1) & \text{if } e > \hat{e}_2 \\ v_t(e;a,k,\eta_\iota) & \text{if } e \in (\hat{e}_{\iota+1}, \hat{e}_\iota] \ \forall \iota \leq \bar{\iota} \\ \frac{\sum_{\iota \geq \bar{\iota}} v_t(e;a,k,\eta_\iota)\cdot \mathbb{P}(\eta_\iota)}{\sum_{\iota < \bar{\iota}} \mathbb{P}(\eta_\iota)} & \text{if } e = 0 \end{cases},$$

which directly implies that the breakeven condition is satisfied:

$$\frac{s_t(e;a,k)}{1+s_t(e;a,k)} \cdot \mathbb{E}[v_t(e;a,k,\eta) \mid \mathcal{B}_t(\eta;e,a,k)] = e \quad \text{for all } e > 0.$$

Step (ii): optimality of $\{e_t(a,k,\eta_\iota)\}_{\iota=1}^n$ Under the equity issuance schedule (6), the value to a firm of type η from choosing issuance level e is given by

$$v_t^0(a,k,\eta;e) = \frac{1}{1+s_t(e;a,k)} \cdot v_t(e;a,k,\eta) = v_t(e;a,k,\eta) - e \cdot [1 + \tau_t(e;a,k,\eta)],$$

where the endogenous lemons wedge $\tau_t(e;a,k,\eta)$ is defined as

$$\tau_t(e;a,k,\eta) = \begin{cases} \frac{v_t(e;a,k,\eta)}{v_t(e;a,k,\eta_1)} - 1 & \text{if } e \geq \hat{e}_2 \\ \frac{v_t(e;a,k,\eta)}{v_t(e;a,k,\eta_\iota)} - 1 & \text{if } e \in (\hat{e}_{\iota+1}, \hat{e}_\iota] \text{ for some } \iota \leq \bar{\iota} \\ \frac{v_t(e;a,k,\eta)}{\frac{\sum_{\iota \geq \bar{\iota}} v_t(e;a,k,\eta_\iota)\cdot \mathbb{P}(\eta_\iota)}{\sum_{\iota \geq \bar{\iota}} \mathbb{P}(\eta_\iota)}} - 1 & \text{if } e = 0 \end{cases}.$$

Now we prove that $e_t(a,k,\eta_\iota)$ is the optimal issuance choice for firms of type η_ι under the issuance schedule $s_t(e; a, k)$, for all $\iota = 1, 2, \ldots, n$, in the following steps. For notational convenience, we write $e \succeq_\eta e'$ ($\succ_\eta$) to denote that issuance level e (strictly) dominates e' for type-η firms under the schedule $s_t(e; a, k)$.

(i) $e_t(a, k, \eta_\iota) \succeq_{\eta_\iota} e$ for all $e \in (\hat{e}_{\iota+1}, \hat{e}_\iota]$.

Proof. For $e \in (\hat{e}_{\iota+1}, \hat{e}_\iota]$, the firm's issuance is priced using its true type: that is, lemons wedge $\tau_t(e; a, k, \eta_\iota) = 0$, and the firm's value is $v_t^0(a, k, \eta_\iota; e) = v_t(e; a, k, \eta_\iota) - e$.

If $e_t(a, k, \eta_\iota) = e_t^*(a, k, \eta_\iota)$, then by definition of $e^*(a, k, \eta_\iota)$

$$v_t^0(a, k, \eta_\iota; e^*) = \max_{e \geq 0} v_t(e; a, k, \eta_\iota) - e \geq \max_{e \in (\hat{e}_{\iota+1}, \hat{e}_\iota]} v_t(e; a, k, \eta_\iota) - e.$$

If instead $e_t(a, k, \eta_\iota) = \bar{e}_t(a, k, \eta_\iota) < e_t^*(a, k, \eta_\iota)$, then $\hat{e}_\iota = \bar{e}_\iota$ and

$$\frac{\partial v_t^0(a, k, \eta_\iota; e)}{\partial e} = \frac{\partial v_t(e; a, k, \eta_\iota)}{\partial e} - 1 > 0$$

for all $e \in (\hat{e}_{\iota+1}, \hat{e}_\iota]$ because of the property (ii) of function $v_t(a, k, \eta)$, which implies that $v_t^0(a, k, \eta_\iota; e_\iota) = v_t^0(a, k, \eta_\iota; \hat{e}_\iota) > v_t^0(a, k, \eta_\iota; e)$ for all $e \in (\hat{e}_{\iota+1}, \hat{e}_\iota]$. ∎

(ii) $e_t(a, k, \eta_\iota) \succeq_{\eta_\iota} e$ for all $e \in (\hat{e}_{\iota+2}, \hat{e}_{\iota+1}]$, i.e., type-$\eta_\iota$ firms have no incentive to reduce issuance and be mistakenly perceived as type-$\eta_{\iota+1}$ firms.

Proof. In this region, the firm's value is

$$v_t^0(a, k, \eta_\iota; e) = v_t(e; a, k, \eta_\iota) - e \cdot \frac{v_t(e; a, k, \eta_\iota)}{v_t(e; a, k, \eta_{\iota+1})}.$$

Since $\frac{\partial v_t(e; a, k, \eta_\iota)}{\partial e} > 1$ for $e \leq \bar{e}_{\iota+1} < e*_\iota$ and $\frac{v_t(e; a, k, \eta_\iota)}{v_t(e; a, k, \eta_{\iota+1})} < 1$, it follows that $v_t^0(a, k, \eta_\iota; e)$ is increasing in e over this region. Hence, for all $e \in (\hat{e}_{\iota+2}, \hat{e}_{\iota+1}]$,

$$v_t^0(a, k, \eta_\iota; e) \leq v_t^0(a, k, \eta_\iota; \hat{e}_{\iota+1}) \leq v_t^0(a, k, \eta_\iota; \bar{e}_{\iota+1}) = v_t^0(a, k, \eta_\iota).$$

∎

(iii) We now show $e_t(a, k, \eta_\iota) \succeq_{\eta_\iota} e$ for all $e \leq \hat{e}_{\iota+1}$, i.e., type-η_ι firms have no incentive to be mistakenly perceived as any higher types.

Proof. For any $e \leq \hat{e}_{\iota+1}$, there exists $\Delta\iota > 0$ such that $e \in (\hat{e}_{\iota+\Delta\iota+1}, \hat{e}_{\iota+\Delta\iota}]$.

From the above proof step (ii), $e_{\iota+\Delta\iota-1} \succeq_{\eta_{\iota+\Delta\iota-1}} e$, i.e., type-$\eta_{\iota+\Delta\iota-1}$ firms have no incentive to deviate from their equilibrium choice $e_{\iota+\Delta\iota-1}$ to issuance $e < e_{\iota+\Delta\iota-1}$. Implied by the single-crossing property (Lemma 4), this preference order will pass to the lower type, i.e., $e_{\iota+\Delta\iota-1} \succ_{\eta_{\iota+\Delta\iota-2}} e$. Again by the above proof in step (ii), because $e_{\iota+\Delta\iota-1} \in (\hat{e}_{\iota+\Delta\iota}, \hat{e}_{\iota+\Delta\iota-1}]$, we have $e_{\iota+\Delta\iota-2} \succ_{\eta_{\iota+\Delta\iota-2}} e_{\iota+\Delta\iota-1} \succ_{\eta_{\iota+\Delta\iota-2}} e$. Recursively applying this argument yields $e_t(a,k,\eta_\iota) \succ_{\eta_\iota} e$. ■

(iv) Likewise, we now prove that $e_t(a,k,\eta_\iota) \succeq_{\eta_\iota} e$ for all $e \in (\hat{e}_\iota, \hat{e}_{\iota-1}]$, i.e., type-$\eta_\iota$ firms have no incentive to issue more and be mistakenly perceived as type-$\eta_{\iota-1}$ firms.

Proof. If $e_\iota = e_\iota^*$, then for $e \in (\hat{e}_\iota, \hat{e}_{\iota-1}]$ we have:

$$v_t^0(a,k,\eta_\iota;e) = v_t(e;a,k,\eta_\iota) - e\frac{v_t(e;a,k,\eta_\iota)}{v_t(e;a,k,\eta_{\iota-1})} < v_t(e;a,k,\eta_\iota) - e < v_t(e_\iota^*;a,k,\eta_\iota) - e_\iota^*.$$

The first inequality follows from the property $\frac{\partial v_t(a,k,\eta)}{\partial \eta} > 0$, the second from the concavity of function $v_t(e;a,k,\eta)$ in issuance e and the definition of e_ι^*.

If $e_\iota = \bar{e}_\iota$, suppose $e \succeq_{\eta_\iota} e_\iota$, then the single-crossing property (Lemma 4) implies $e \succ_{\eta_{\iota-1}} e_\iota$. Since type $\eta_{\iota-1}$ is indifferent between $e_\iota = \bar{e}_\iota$ and $e_{\iota-1}$ by construction (non-mimicking condition (37)), then we will have $e \succ_{\eta_{\iota-1}} e_{\iota-1}$, which contradicts with the prior result that $e_{\iota-1}$ dominates all $e \in (\hat{e}_\iota, \hat{e}_{\iota-1}]$ for type $\eta_{\iota-1}$. ■

(v) Finally, we show $e_t(a,k,\eta_\iota) \succeq_{\eta_\iota} e$ for all $e > \bar{e}_\iota$, i.e., type-η_ι firms have no incentive to issue more and be mistakenly perceived as lower types.

Proof. Suppose there exists an $e > \bar{e}_\iota$ such that $e \succ_{\eta_\iota} e_\iota$, then there exists an $\Delta\iota > 0$ such that $e \in (\hat{e}_{\iota-\Delta\iota+1}, \hat{e}_{\iota-\Delta\iota}]$. If e dominates e_ι for type-η_ι firms, the single-crossing property implies $e \succ_{\eta_{\iota-1}} e_\iota^*$. In the meantime, $e_\iota \neq e_\iota^*$ because

$$v_t^0(a,k,\eta_\iota;e_\iota^*) = \max_{e\geq 0} v_t(e;a,k,\eta_\iota) - e > v_t(e;a,k,\eta_\iota) - e\cdot\frac{v_t(e;a,k,\eta_\iota)}{v_t(e;a,k,\eta_{\iota-\Delta\iota})} = v_t^0(a,k,\eta_\iota;e),$$

and we must have $e_\iota = \bar{e}_\iota$. Because type-$\eta_{\iota-1}$ firms are indifferent between $\bar{e}_\iota$ and $e_{\iota-1}$ (non-minicking condition (37)), we have $e \succ_{\eta_{\iota-1}} e_{\iota-1}$. Repeating the above logic recursively, we conclude $e \succ_{\eta_{\iota-\Delta\iota}} e_{\iota-\Delta\iota}$, contradicting the earlier result that $e_{\iota-\Delta\iota} \succeq_{\eta_{\iota-\Delta\iota}} e$ for all $e \in (\hat{e}_{\iota-\Delta\iota+1}, \hat{e}_{\iota-\Delta\iota}]$. ■

(vi) Combining steps (i), (iii), and (v), we conclude that $e_t(a,k,\eta_\iota)$ is the optimal issuance level for type-η_ι firms under $s_t(e;a,k)$.

Step (iii): consistency of belief $\mathcal{B}_t(\eta;e,a,k)$ Following the specification of the D1 criterion, we first prove that the type most likely to deviate to a signal $e \in (\hat{e}_{\iota+1}, \hat{e}_\iota)$ is η_ι. For a given signal e, we define

$$\bar{s}_t(e;a,k,\eta_\iota) \equiv \frac{v_t(e;a,k,\eta_\iota)}{v_t^0(a,k,\eta_\iota)} - 1$$

as the upper bound on the number of new shares required to make type-η_ι firms indifferent between deviating to e and staying at their equilibrium choice. We then prove the following results:

(i) $\bar{s}_t(e;a,k,\eta_\iota) > \bar{s}_t(e;a,k,\eta_{\iota-1})$ for any $e < \hat{e}_\iota$, i.e., type-η_ι firms are more likely to deviate to e than type-$\eta_{\iota-1}$ firms.

Proof. Define $\Delta\bar{s}(e) \equiv \bar{s}_t(e;a,k,\eta_\iota) - \bar{s}_t(e;a,k,\eta_{\iota-1})$, then

$$\Delta\bar{s}'(e) = \frac{\partial v_t(e;a,k,\eta_\iota)/\partial e}{v_t^0(a,k,\eta_\iota)} - \frac{\partial v_t(e;a,k,\eta_{\iota-1})/\partial e}{v_t^0(a,k,\eta_{\iota-1})}.$$

Since $\partial^2 v_t(e;a,k,\eta)/\partial e\,\partial\eta \leq 0$ implies that $\partial v_t(e;a,k,\eta_\iota)/\partial e \leq \partial v_t(e;a,k,\eta_{\iota-1})/\partial e$, and $v_t^0(a,k,\eta_\iota) > v_t^0(a,k,\eta_{\iota-1})$ by the preparatory result, it follows that $\Delta\bar{s}'(e) < 0$, i.e., $\Delta\bar{s}(e)$ is decreasing in e.

When $\hat{e}_\iota = \bar{e}_\iota$, then the non-mimicking condition (37) implies

$$\begin{aligned}\Delta\bar{s}(\hat{e}_\iota) = \Delta\bar{s}(\bar{e}_\iota) =& \frac{v_t(\bar{e}_\iota;a,k,\eta_\iota)}{v_t^0(a,k,\eta_\iota)} - \frac{v_t(\bar{e}_\iota;a,k,\eta_{\iota-1})}{v_t^0(a,k,\eta_{\iota-1})} \\ =& \frac{v_t(\bar{e}_\iota;a,k,\eta_\iota)}{v_t(\bar{e}_\iota;a,k,\eta_\iota) - \bar{e}_\iota} - \frac{v_t(\bar{e}_\iota;a,k,\eta_{\iota-1})}{v_t(\bar{e}_\iota;a,k,\eta_{\iota-1}) - \bar{e}_\iota \cdot \frac{v_t(\bar{e}_\iota;a,k,\eta_{\iota-1})}{v_t(\bar{e}_\iota;a,k,\eta_\iota)}} = 0.\end{aligned}$$

Similarly, when $\hat{e}_\iota < \bar{e}_\iota$, the definition of $\hat{e}_\iota$ implies $\Delta\bar{s}(\hat{e}_\iota) = 0$. Since $\Delta\bar{s}(e)$ is strictly decreasing and equals zero at $e = \hat{e}_\iota$, it must be that $\Delta\bar{s}(e) > 0$ for all $e < \hat{e}_\iota$. ■

(ii) $\bar{s}_t(e;a,k,\eta_\iota) > \bar{s}_t(e;a,k,\eta_{\iota+1})$ for any $e > \hat{e}_{\iota+1}$, i.e., type-η_ι firms are more likely to deviate to e than type-$\eta_{\iota+1}$ firms.

Proof. Define $\Delta\bar{s}(e) \equiv \bar{s}_t(e; a, k, \eta_\iota) - \bar{s}_t(e; a, k, \eta_{\iota+1})$. Then

$$\Delta\bar{s}'(e) = \frac{\partial v_t(e; a, k, \eta_\iota)/\partial e}{v_t^0(a, k, \eta_\iota)} - \frac{\partial v_t(e; a, k, \eta_{\iota+1})/\partial e}{v_t^0(a, k, \eta_{\iota+1})}.$$

Since $\partial v_t(e; a, k, \eta)\partial e$ is weakly decreasing in η and $v_t^0(a, k, \eta)$ is strictly increasing in η, we have $\Delta\bar{s}'(e) > 0$. By the definition of $\hat{e}_{\iota+1}$, we have $\Delta\bar{s}(\hat{e}_{\iota+1}) = 0$, and thus $\Delta\bar{s}(e) > 0$ for any $e > \hat{e}_{\iota+1}$. ■

(iii) $\bar{s}_t(e; a, k, \eta_\iota) > \bar{s}_t(e; a, k, \eta_{\iota'})$ for any $e \in (\hat{e}_{\iota+1}, \hat{e}_\iota)$ and $\iota' \neq \iota$, i.e., type-η_ι firms are more likely to deviate to e than any other types.

Proof. From step (i), we know $\bar{s}_t(e; a, k, \eta_\iota) > \bar{s}_t(e; a, k, \eta_{\iota-1})$, and since $e < \hat{e}_\iota < \hat{e}_{\iota-1}$, we also have $\bar{s}_t(e; a, k, \eta_{\iota-1}) > \bar{s}_t(e; a, k, \eta_{\iota-2})$. Repeating this argument leads to

$$\bar{s}_t(e; a, k, \eta_\iota) > \bar{s}_t(e; a, k, \eta_{\iota-1}) > \cdots > \bar{s}_t(e; a, k, \eta_1).$$

Similarly, from step (ii), we obtain

$$\bar{s}_t(e; a, k, \eta_\iota) > \bar{s}_t(e; a, k, \eta_{\iota+1}) > \cdots > \bar{s}_t(e; a, k, \eta_{\bar{\iota}}).$$

Together, this proves that type-η_ι is strictly more likely to deviate to $e \in (\hat{e}_{\iota+1}, \hat{e}_\iota)$ than any other type. ■

A.1.3 Non-existence of Pooling Equilibrium

Finally, we show that a pooling equilibrium with strictly positive equity issuance does not exist. The intuition is straightforward: a high-type firm has an incentive to slightly reduce its issuance in order to distinguish itself from lower types that are attempting to pool at the same level. We formalize this insight in the following proposition.

***Proposition* 4** (Non-Existence of Pooling Equilibrium)**.** *Given a post-issuance value function $v_t(e; a, k, \eta)$ and a fixed set of publicly observable states (a, k), there does not exist an equilibrium in which multiple firm types choose the same strictly positive equity issuance level $e^{pool} > 0$.*

Proof. Directly following from the single-crossing property (Lemma 4), under any share schedule, higher-type firms never issue more than lower-type firms. Hence, in any equilibrium, the issuance levels must satisfy $e_1 \geq e_2 \geq \cdots \geq e_N$. Let $\mathcal{T}^{pool} \equiv \{\eta_\iota\}_{\iota=\underline{n}}^{\bar{n}}$ denote the set of types issuing at $e^{pool} > 0$. Then we have $e_\iota > e^{pool}$ for all $\iota < \underline{n}$ and $e_\iota < e^{pool}$ for all $\iota > \bar{n}$. The investment fund's belief at e^{pool}, based on Bayes' rule, is given by

$$\mathcal{B}(\eta; e^{pool}, a, k) = \frac{\sum_{\tilde{\eta} \in \mathcal{T}^{pool}} \mathbb{1}\{\eta = \tilde{\eta}\} \cdot \mathbb{P}[\tilde{\eta}]}{\sum_{\tilde{\eta} \in \mathcal{T}^{pool}} \cdot \mathbb{P}[\tilde{\eta}]}.$$

The type-$\eta_{\bar{n}}$ firm's value at e^{pool} is

$$v^{pool}(\eta_{\bar{n}}) = v_t(e^{pool}; a, k, \eta_{\bar{n}}) - e^{pool} \cdot \frac{v_t(e^{pool}; a, k, \eta_{\bar{n}})}{\mathbb{E}[v_t(e^{pool}; a, k, \eta) \mid \eta \in \mathcal{T}^{pool}]}.$$

Now consider a deviation to an off-equilibrium issuance level $e^{pool} - \Delta e$ for an infinitesimal $\Delta e > 0$. As in the proof of belief consistency in the separating equilibrium, the single-crossing property implies that type-$\eta_{\bar{n}}$ is strictly more likely to deviate than any lower-type firm. By the D1 criterion, the investment fund's belief at $e^{pool} - \Delta e$ must assign zero probability to types below $\eta_{\bar{n}}$. Let $\mathcal{T}^{dev}$ denote the set of types assigned positive probability at this deviation; then the lowest possible type in $\mathcal{T}^{dev}$ is at least $\eta_{\bar{n}}$.

The deviating value for type-$\eta_{\bar{n}}$ is

$$v^{dev}(\eta_{\bar{n}}) = v_t(e^{pool} - \Delta e; a, k, \eta_{\bar{n}}) - (e^{pool} - \Delta e) \cdot \frac{v_t(e^{pool} - \Delta e; a, k, \eta_{\bar{n}})}{\mathbb{E}[v_t(e^{pool} - \Delta e; a, k, \eta) \mid \eta \in \mathcal{T}^{dev}]}.$$

Taking the limit as $\Delta e \to 0$, we obtain

$$\begin{aligned}\lim_{\Delta e \to 0} v^{dev}(\eta_{\bar{n}}) &= v_t(e^{pool}; a, k, \eta_{\bar{n}}) - e^{pool} \cdot \frac{v_t(e^{pool}; a, k, \eta_{\bar{n}})}{\mathbb{E}[v_t(e^{pool}; a, k, \eta) \mid \eta \in \mathcal{T}^{dev}]} \\ &> v_t(e^{pool}; a, k, \eta_{\bar{n}}) - e^{pool} \cdot \frac{v_t(e^{pool}; a, k, \eta_{\bar{n}})}{\mathbb{E}[v_t(e^{pool}; a, k, \eta) \mid \eta \in \mathcal{T}^{pool}]} = v^{pool}(\eta_{\bar{n}}),\end{aligned}$$

since $\mathbb{E}[v_t(e^{pool}; a, k, \eta) \mid \eta \in \mathcal{T}^{dev}] > \mathbb{E}[v_t(e^{pool}; a, k, \eta) \mid \eta \in \mathcal{T}^{pool}]$. This inequality shows that the highest type firm within the pool can always reduce their issuance by an infinitesimal amount to trigger a discretely better pricing from the investment fund. Therefore, no belief consistent with the D1 criterion can support e^{pool} as a pooling issuance choice in equilibrium.

■

A.2 Special Case From Main Text

We now turn to the special case with one-shot private information and no exogenous equity issuance costs featured in the main text.

A.2.1 Proof for Proposition 1

Step 1: Pricing of new shares. Under full information, investment funds' breakeven condition (6) becomes

$$\frac{s_t(e;a,k,\eta)}{1+s_t(e;a,k,\eta)}\, v_t(e;a,k,\eta) \;=\; e.$$

Solving for shares demand $s_t(e;a,k,\eta)$ gives

$$s_t(e;a,k,\eta) = \frac{e}{v_t(e;a,k,\eta)-e}.$$

Step 2: Firm's problem. Insert the solution for s_t back into the objective (2) to obtain

$$v_t^0(a,k,\eta) \;=\; \max_{e\geq 0} \frac{v_t(e;a,k,\eta)}{1+s_t(e;a,k,\eta)} \;=\; \max_{e\geq 0}\big[v_t(e;a,k,\eta)-e\big]. \tag{52}$$

Then substitute the flow-of-funds identity (4),

$$d = A_t(a)k + e - x - \psi(x),$$

into the continuation value (3) and stack the two maximizations. The firm's problem can be written as

$$v_t^0(a,k,\eta) = \max_{x,e\geq 0}\Big\{A_t(a)k + e - x - \psi(x) + \mathbb{E}_t\big[\Lambda_{t,t+1}\{\xi k' + (1-\xi)v_{t+1}^0(a',k',\eta')\}\big] - e\Big\}$$

subject to the law of motion for capital (5) and the non-negativity constraint on dividends, $A_t(a)k+e-x-\psi(x)\geq 0$. We guess that this constraint is not binding and then verify that there exist optimal financing policies which satisfy it.

The two terms $\pm e$ cancel, so the choice variable e drops out of the objective. Because e only appears in the non-negativity constraint $e\geq 0$ and in no other restriction, the remaining optimization is over x alone. Plugging in the law of motion for capital k' (5) yields

$$v_t^0(a,k,\eta) = \max_{x}\Big\{A_t(a)k - x - \psi(x) + \mathbb{E}_t\big[\Lambda_{t,t+1}\{\xi(\eta(1-\delta)k+x) + (1-\xi)v_{t+1}^0(a',\eta(1-\delta)k+x,\eta')\}\big]\Big\}.$$

To show that the value function has the additively separable form (9), plug in the form (9) to the RHS of the Bellman operator above and collect terms to get

$$\begin{aligned} v_t^0(a,k,\eta) = {} & \big(A_t(a) + \mathbb{E}_t[\Lambda_{t,t+1}\{\xi\eta(1-\delta) + (1-\xi)\eta(1-\delta)Q_{t+1}^k(a',\eta')\}]\big)\, k \\ & + \max_x\Big\{-x-\psi(x) + \mathbb{E}_t\left[\Lambda_{t,t+1}\{\xi + (1-\xi)Q_{t+1}^k(a',\eta')\}\right] x\Big\} + (1-\xi)\mathbb{E}_t[\Lambda_{t,t+1}Q_{t+1}^x(a')]. \end{aligned} \tag{53}$$

Since the space of additively separable functions is closed and compact, and the Bellman equation is a contraction mapping, its fixed point must preserve additive separability. Hence, collecting terms in (53) at the optimum establishes the additive separability of the pre-issuance value function (9). The additive separability of the post-issuance value function (10) follows from (52).

Step 3: Optimal investment. Using the additively separable functional form, the first order condition for optimal investment in (53) is

$$1 + \psi'(x) = \mathbb{E}_t \left[\Lambda_{t,t+1}(\xi + (1-\xi) Q^k_{t+1}(a', \eta') \right].$$

Inverting this expression yields (11) in the main text.

Step 4: Dividends and external finance. Given $x^*_t(a,k,\eta)$, the flow-of-funds constraint yields a set of dividend and equity issuance policies consistent with $e^*_t(a,k,\eta) \geq 0$ and $d^*_t(a,k,\eta) \geq 0$. Taking the limit $\varphi_1 \to 0$ implies that we select the policy which minimizes equity issuance.

■

A.2.2 Proof for Lemma 1

Under one-shot asymmetric information, the pre-issuance value function in period $t+1$ reverts to the full information case, $v^0_{t+1}(a', k', \eta') = Q^k_{t+1}(a', \eta')k' + Q^x_{t+1}(a')$. Given this value, the post-issuance value function in period t solves

$$v_t(e; a, k, \eta) = \max_x A_t(a)k + e - x - \psi(x) + \mathbb{E}_t \left[\Lambda_{t,t+1}(\xi k' + (1-\xi)(Q^k_{t+1}(a', \eta')k' + Q^x_{t+1}(a'))) \right] \tag{54}$$

subject to the non-negativity constraint on dividends $A_t(a)k + e - x - \psi(x) \geq 0$ and the law of motion for capital $k' = \eta(1-\delta)k + x$.

We assign the Lagrange multiplier $\lambda_t(e; a, k, \eta)$ to the non-negativity constraint on dividends and plug in the law of motion for capital to the objective (54). The first-order condition for investment is

$$(1 + \psi'(x))(1 + \lambda_t(e; a, k, \eta)) = \mathbb{E}_t \left[\Lambda_{t,t+1}(\xi + (1-\xi) Q^k_{t+1}(a', \eta')) \right]. \tag{55}$$

If the full-information investment policy is feasible, i.e. satisfies the non-negativity constraint on dividends, then it solves the FOC (55) and is optimal. In this case, the problem reduces to the full information problem, establishing part (i) of the proposition.

If the full-information policy is not feasible, then the non-negativity constraint on dividends in binding, giving $x + \psi(x) = A_t(a)k + e$. This equation defines the policy function for investment in this case. Plugging $d = 0$ into the objective function (54) gives the post-issuance value (15), establishing part (ii) of the proposition. ■

A.2.3 Proof for Proposition 2

Proposition 2 is the version of the general Propositions 3 and 4 applied to the special case. Therefore, we only need to show that the post-issuance value function satisfies Properties (i)—(iii) from Section A.1.

(i) As shown in Lemma 1, the firm's post-issuance value function in the special case is

$$v_t(e; a, k, \eta) \tag{56}$$
$$= \begin{cases} \widetilde{Q}_t^k(a) \times \eta(1-\delta)k + \widetilde{Q}_t^k(a) \times x_t(e; a, k) + \widetilde{Q}_t^x(a) & \text{if } e < x_t^*(a) + \psi(x_t^*(a)) - A_t(a)k \\ Q_t^k(a, \eta) \cdot k + Q_t^x(a) + e & \text{if } e \geq x_t^*(a) + \psi(x_t^*(a)) - A_t(a)k. \end{cases}$$

Taking partial derivative with respect to η and e yields $\frac{\partial v_t(e;a,k,\eta)}{\partial \eta} = (1-\delta)k\widetilde{Q}_t^k(a)$ and

$$\frac{\partial v_t(e; a, k, \eta)}{\partial e} = \begin{cases} \widetilde{Q}_t^k(a) \times [1 + \psi'(x_t(e; a, k))]^{-1} & \text{if } e < x_t^*(a) + \psi(x_t^*(a)) - A_t(a)k\ , \\ 1 & \text{if } e \geq x_t^*(a) + \psi(x_t^*(a)) - A_t(a)k; \end{cases}$$

which implies that the property (i) holds in the special case.

(ii) When $e < x_t^*(a) + \psi(x_t^*(a)) - A_t(a)k$, i.e., issuance is not high enough to fully cover the full-information investment, the marginal value of equity funding satisfies

$$\frac{\partial v_t(e; a, k, \eta)}{\partial e} > \widetilde{Q}_t^k(a)\psi'(x_t^*(a)) = 1$$

because $x_t(e; a, k) < x_t^*(a)$ and $\psi''(x) > 0$; and

$$\frac{\partial^2 v_t(e; a, k, \eta)}{\partial e^2} = -\widetilde{Q}_t^k(a) \times \psi''(x_t(e; a, k)) \times [1 + \psi'(x_t(e; a, k))]^{-3} < 0.$$

Therefore, the property (ii) also holds in the special case, and the source of it is the convexity of investment adjustment cost.

(iii) From (56), we have $\frac{\partial^2 v_t(e;a,k,\eta)}{\partial e \partial \eta} = 0$, implying that the property (iii) also holds in the special case. This property directly comes from the additive separability of the value function, which is a result of our setup of constant returns to scale technology.

A.2.4 Proof for Lemma 3

Denote the two types of capital η_L and η_H with probabilities p_L and p_H. In this special case, both types issue positive equity in the afternoon, so we can compute the afternoon price as $p_t^a(a,k,\eta) = \frac{e_t(a,k,\eta)}{s_t(a,k,\eta)}$. From the investment funds' breakeven condition, we can write this ratio as

$$p_t^a(a,k,\eta) = \frac{1}{1+s_t(a,k,\eta)} v_t(e_t(a,k,\eta);a,k,\eta).$$

The no-mimicking condition (16) for the low type implies that its afternoon price is

$$p_t^a(a,k,\eta_L) = \frac{1}{1+s_t(a,k,\eta_L)} v_t(e_t(a,k,\eta_L);a,k,\eta_L) = \frac{1}{1+s_t(a,k,\eta_H)} v_t(e_t(a,k,\eta_H);a,k,\eta_L).$$

Putting these results together gives the morning price

$$p_t^m(a,k) = \frac{1}{1+s_t(a,k,\eta_H)} \left[p_L \times v_t(e_t(a,k,\eta_H);a,k,\eta_L) + p_H \times v_t(e_t(a,k,\eta_H);a,k,\eta_H)\right]. \tag{57}$$

We now use the additively separable post-issuance value function to simplify (57). In particular, plugging (15) into (57) and collecting terms gives

$$p_t^m(a,k) = \frac{1}{1+s_t(a,k,\eta_H)} \left[x_t(e_t(a,k,\eta_H);a,k)\widetilde{Q}_t^k(a) + \widetilde{Q}_t^x(a)\right] + \frac{1}{1+s_t(a,k,\eta_H)} \left[(1-\delta)k\widetilde{Q}_t^k(a)\overline{\eta}\right].$$

The level difference between the afternoon and morning price is then

$$p_t^a(a,k,\eta) - p_t^m(a,k) = \frac{1}{1+s_t(a,k,\eta_H)}(1-\delta)k\widetilde{Q}_t^k(a)(\eta-\overline{\eta}).$$

Dividing by the morning price $p_t^m(a,k)$ and simplifying gives

$$\frac{p_t^a(a,k,\eta) - p_t^m(a,k)}{p_t^m(a,k)} = \frac{(1-\delta)k\widetilde{Q}_t^k(a)(\eta-\overline{\eta})}{(1-\delta)k\widetilde{Q}_t^k(a)\overline{\eta} + x_t(e_t(a,k,\eta_H);a,k)\widetilde{Q}_t^k(a) + \widetilde{Q}_t^x(a)}.$$

Finally, divide the numerator and denominator by $(1-\delta)k\widetilde{Q}_t^k(a)$ and use the fact that the percentage change approximately equals the log difference to arrive at equation (22) in the main text.

B Data Appendix

B.1 Data Cleaning

To construct our sample of SEOs from SDC Platinum, we apply a series of filters to ensure data quality and relevance.

(i) We retain only those offerings by the U.S. firms listed on the NASDAQ, New York Stock Exchange (NYSE), and American Stock Exchange (AMEX).

(ii) We exclude offerings from firms in the financial sectors (SIC codes 6000-6999), utility sectors (SIC codes 4900-4999), and quasi-governmental sectors (SIC codes 9000-9999).

(iii) We exclude initial public offerings (IPOs).

(iv) We limit the sample to SEOs completed between the first quarter of 1985q1 and the fourth quarter of 2018q4.

(v) We restrict the sample to offerings involving only the issuance of primary common shares, because the proceeds of secondary shares issuance will not go to the firm.

(vi) We remove offerings in which the proportion of new shares issued exceeds the 95th percentile (approximately 54% of initial shares) because these offerings typically involve organizational changes in the firms.

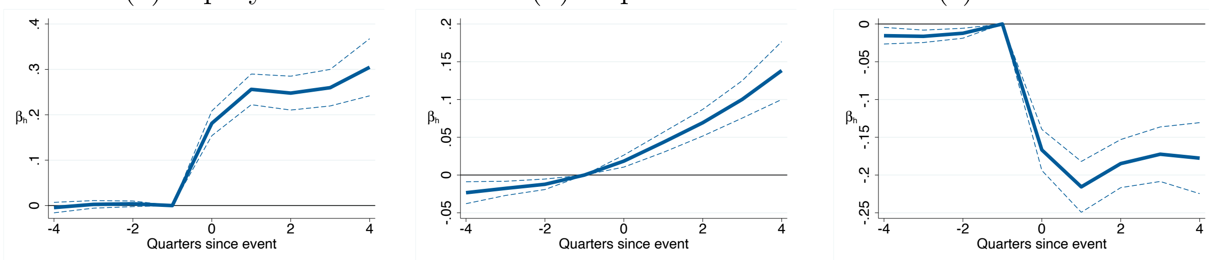
We extract the daily stock price history from CRSP for each SEO in our sample. Based on these stock price histories, we then construct the stock price change by summing the stock returns during the first two days of the issuance event for each offering (as defined in Section 4.1). To mitigate the influence of outliers, we further exclude observations with stock price changes during the issuance event falling outside the 0.5 to 99.5 percentile range.

B.2 Dynamics of Investment Around Equity Issuance Events

We now show that firms use the majority of the proceeds from new equity to invest in new capital. For this, we use Compustat data, following the cleaning procedure from Ottonello

Figure B.1: Firms' Use of Proceeds

(a) Equity (b) Capital (c) Net Debt



Notes: Reports the coefficient β_h from estimating (58). See the main text for the definition of variables. Standard errors are two-way clustered by firm and quarter. Dashed lines report 90% confidence intervals.

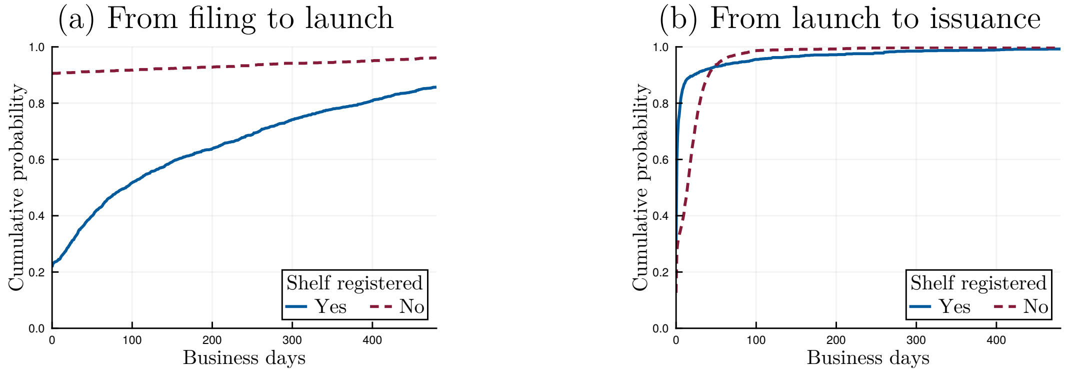
and Winberry (2020), extended to cover the period through 2019. Using these data, we estimate local projections à la Jorda (2005):

$$(y_{jt+h} - y_{jt-1})/\text{assets}_{jt-1} = \alpha_{jh} + \alpha_{sth} + \beta_h \cdot \text{issuance}_{jt} + \Gamma'_h X_{jt-1} + \varepsilon_{jt+h}, \tag{58}$$

where y_{jt} denotes the book value of equity, capital, or net debt for firm j in quarter t, and assets_{jt} is its book value of total assets. We define capital as total assets minus cash and receivables; net debt as total debt plus payables, minus cash and receivables; and equity as total assets minus total liabilities. The variable issuance_{jt} is an indicator equal to one if firm j experiences an issuance event (as defined in Section 4.1) in quarter t. The specification includes firm-by-horizon fixed effects α_{jh} and sector-by-time-by-horizon fixed effects α_{sth}. The vector X_{jt-1} is a set of firm-level controls containing lagged sales growth, firm size, current assets as a share of total assets, and an indicator for fiscal quarter, following Ottonello and Winberry (2020).

Panel (a) of Figure B.1 shows that firms increase their book equity by 25% of assets over the four quarters following the announcement. Panel (b) shows that capital increases by 14% over the same window, suggesting that firms use the majority of the proceeds to finance investment in the first year after issuance. Panel (c) shows that firms also use most of the remaining proceeds to reduce net debt. These results are robust to excluding controls or to controlling directly for the level of issuance.

Figure B.2: Distribution of the Time Gap between Different Types of Event Dates



Notes: Panel (a) presents the cumulative density function (CDF) of the number of business days between the date when firms initially file to SEC and the date when firms actually launch the issuance. Panel (b) presents the CDF of the number of business days between the date when firms launch the issuance and the date when the issuance is complete. In both panels, the solid lines refer to the CDF for the shelf-registered issuance and the red dash lines refer to the CDF for the non-shelf-registered issuance.

B.3 Additional Results About High-Frequency Price Changes

We now provide additional empirical results referenced in Section 4.1 of the main text.

Different types of event dates As described in Section 4.1, each issuance event has three types of relevant event dates: the filing date, the launch date, and the issuance date. Figure B.2 summarizes the distribution of the time elapsed between these three types of dates. For non-shelf registered events, almost all of them launch the issuance on the same day when they first file with SEC. After launch, a third of the events complete the issuance within a week and more than 90% of them complete the issuance within a quarter. For shelf-registered events, the gap between their launch date and filing date is much larger because they don't need to commit to a specific time to complete the issuance.

Alternative measures of price drops Table B.1 shows that our measurement of the average price drop upon issuance is robust to a number of alternative choices. First, we construct the stock price change based on abnormal daily returns estimated using the Fama and French (2015) five-factor model. We examine two estimation windows for individual stocks' α and βs: from 160 to 10 business days before the issuance event, and from 10 to 160 business days after. Under both specifications, the resulting average price drops are similar to our baseline.Second, we calculate stock price changes by summing daily stock

Table B.1
Alternative Measurement of the Price Drops

	Baseline	Filtered		All events	Shelf-registered	
		Pre-event	Post-event		Yes	No
Obs	3178	3178	3178	3178	1854	1304
Average						
weighted, cross-time	-3.54	-3.76	-3.31	-4.30	-2.97	-4.17
weighted, pooled	-3.84	-4.05	-3.65	-4.90	-3.29	-4.88
non-weighted, pooled	-3.45	-3.67	-3.26	-4.26	-3.06	-4.05
Standard deviation	7.63	7.39	7.34	10.12	7.32	7.96
Percentiles						
10%	-12.41	-12.39	-11.63	-16.23	-11.12	-13.75
25%	-6.81	-6.96	-6.64	-9.34	-6.22	-8.12
50%	-2.59	-2.79	-2.46	-3.35	-2.17	-3.00
75%	0.57	0.35	0.59	1.80	0.59	0.45
90%	4.27	3.95	4.21	7.11	4.22	4.28

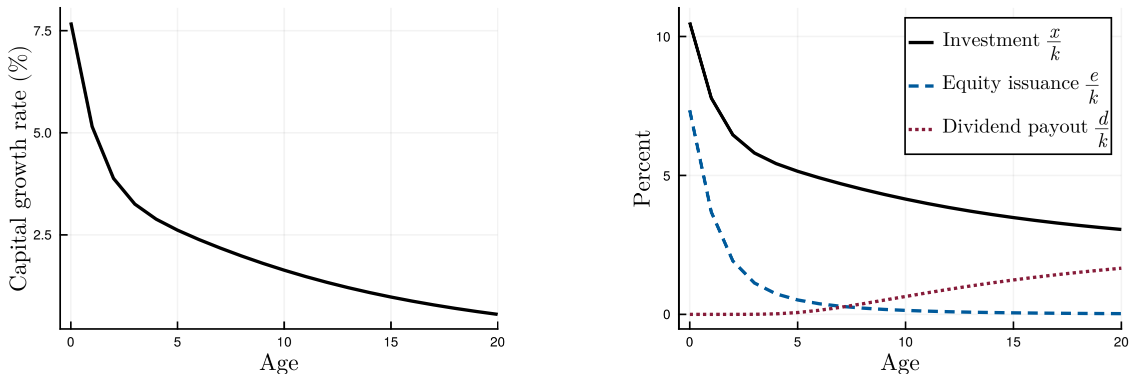
Notes: The "*Baseline*" column reports descriptive statistics of total stock price changes over the two-day window beginning on our chosen event date, as described in Section 4.1. The "*Filtered*" columns present descriptive statistics of stock price changes based on abnormal daily returns estimated using the Fama-French five-factor model. The labels "*Pre-event*" and "*Post-event*" indicate the time windows, 10th to 160th business days before or after our issuance event date, used to estimate each stock's α and β coefficients. The "*All events*" column reports statistics based on total stock price changes across all three types of event dates for each issuance. The "*Shelf-registered*" columns separately report baseline statistics for shelf-registered and non-shelf-registered issuances. The table includes averages based on three calculation methods. "*Weighted, cross-time*" refers to the time-series average of quarterly mean price drops, weighted by the ratio of newly issued shares to existing shares. "*Weighted, pooled*" refers to the cross-sectional average over the entire sample, also weighted by the ratio of newly issued shares to existing shares. "*Non-weighted, pooled*" is the simple (unweighted) average across all observations. All other reported statistics are based on the full sample and are unweighted. The unit of the reported statistics is percent.

returns across all event dates for each issuance, not just the dates used in the main text.[24] Under this approach, the average price drop is nearly 1% larger than our baseline estimate, primarily due to the inclusion of returns on the issuance date. Finally, we split the sample into shelf-registered and non-shelf-registered issuances. On average, non-registered offerings experience larger price drops, consistent with the notion that firms can better time the market when using shelf registration.

[24]Multiple event dates often overlap within one issuance; we include each overlapping day only once to avoid double counting.

Figure C.1: Life-cycle dynamics in our model

(a) Growth rate (b) Financing flows



Notes: Panel (a) plots the average quarterly growth rate of firms' capital at different ages. Panel (b) plots the average quarterly investment and financing flow, all normalized by their capital stock, across firms of different ages.

C Additional Quantitative Results

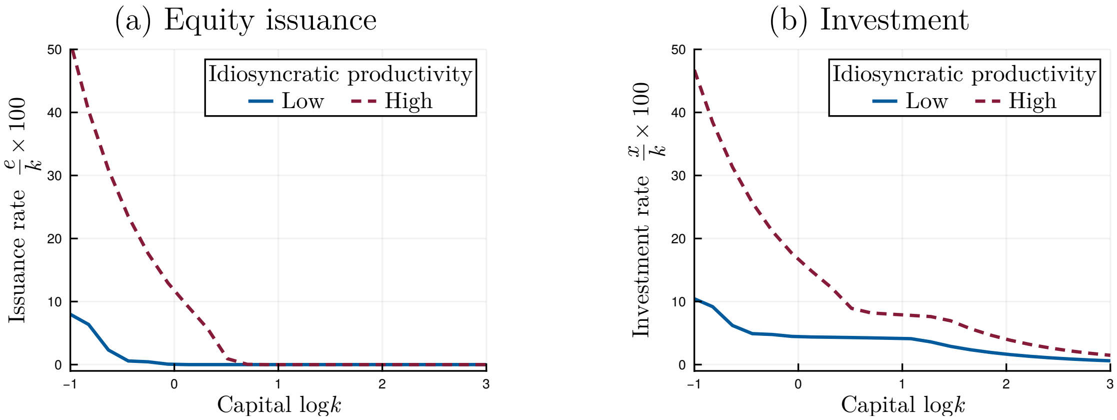
In this appendix, we present three additional sets of quantitative results from our model. First, we analyze the steady state firm dynamics in our calibrated model. Second, we show that the model generates stock price runups before equity offerings, as in the data. Finally, we provide additional results about the aggregate lemons shocks.

C.1 Firm Dynamics

Figure C.1 shows that young firms grow faster and rely more on external financing than old firms in our model, as in the data. These dynamics also resemble the typical lifecycle dynamics produced by heterogeneous firm models with decreasing returns to scale. In those models, young firms grow faster than average because they have a higher marginal product of capital. In our model, the marginal product of capital is independent of size due to constant returns to scale. Nevertheless, young firms grow faster than average because the adjustment costs in our model limit the level of investment, which is a larger fraction of young firms because they are small.

Figure C.2 shows how observable idiosyncratic productivity shocks affects the firm's equity issuance and investment decisions. Since productivity is persistent, a positive productivity shock raises the expected marginal product of capital, increasing desired investment.

Figure C.2: Average Policies across Firms with Different Idiosyncratic Productivities



Notes: Panel (a) shows the average equity issuance of firms in the steady state, conditional on idiosyncratic productivity and capital stock but averaged over different capital quality levels, i.e., $\frac{\mathbb{E}[e_t(a,k,\eta)|(a,k)]}{k}$. Similarly, Panel (b) presents the average investment policies in the steady state, given by $\frac{\mathbb{E}[x_t(a,k,\eta)|(a,k)]}{k}$. In both panels, "*Low*" denotes the median level of idiosyncratic productivity, while "*High*" corresponds to the 90th percentile.

Small firms, who do not have enough revenues to self-finance this increased investment, must raise new equity to finance this investment. Note that this feature implies that firms who experience positive productivity shocks are more likely to issue equity and, therefore, are more likely to appear in the sample of issuing firms.

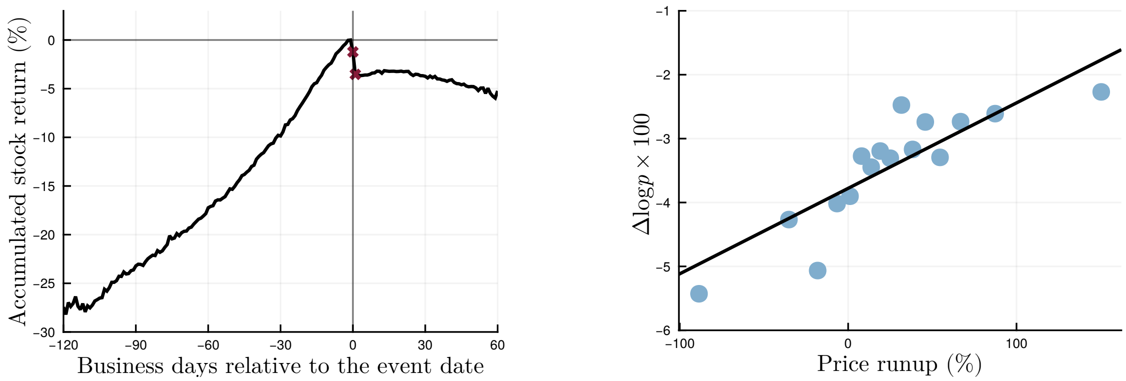
C.2 Stock Price Runups in the Model vs. the Data

We now show that our model generates an empirically realistic stock price runup prior to issuing equity. We also show that the size of the runup is positive correlated with the average price drop upon issuance in both the model and the data.

Data The left panel of Figure C.3 shows the average time series of stock prices in our data. On average, issuing firms experience a 27% increase in stock price over the 120 business days leading up to the offering, consistent with existing estimates in the corporate finance literature (e.g., Masulis and Korwar, 1986). The right panel shows a binscatter of the size of this runup with the magnitude of the price drop upon issuance; the two are positively correlated, i.e. a larger price runup predicts a smaller price drop.

Figure C.3: Stock Price Runup Before Equity Issuance in the Data

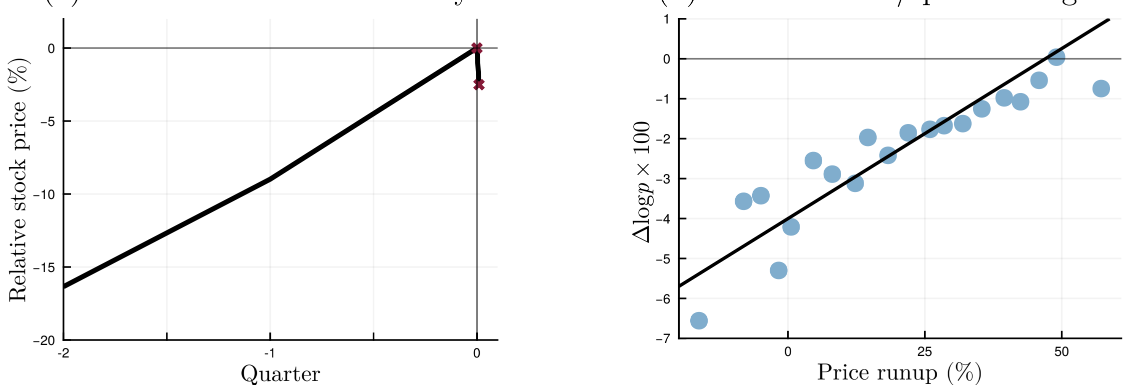
(a) Trajectory of stock prices (b) Correlation with price drops



Notes: Panel (a) depicts the trajectory of average stock price relative to the day before issuance date. Panel (b) binscatter plots the price drop against the total price runup during the 120 business days before the issuance event date for the offerings in our sample.

Figure C.4: Stock Price Runup before Equity Issuance in the Model

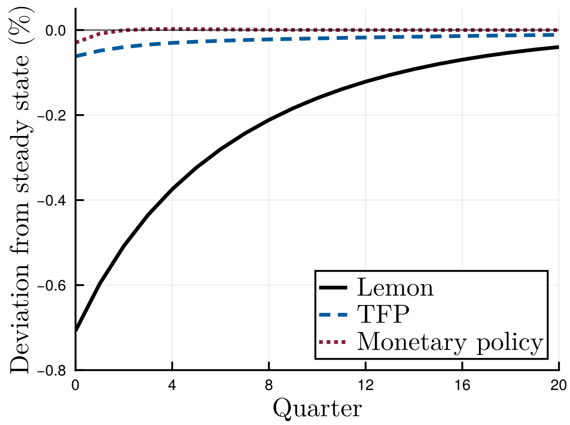
(a) Accumulated return history (b) Correlation w/ price changes



Notes: Panel (a) depicts the trajectory of average stock price relative to the pre-issuance price based on a simulated panel of firms in our model. Panel (b) binscatter plots the price drop against the total price runup during the two quarters before the equity issuance in the same simulated data.

Model Figure C.4 shows that our model replicates both of these patterns. The left panel the stock prices of issuing firms increase by approximately 16% during the two quarters—roughly 120 business days—leading up to the equity issuance. As discussed in the main text, this result reflects positive selection into equity issuance according to the history of observable idiosyncratic productivity shocks (see also Figure C.2). The right panel shows that a larger runup also predicts a smaller price drop. As discussed in the main text, these results reflect positive selection into equity issuance. In particular, firms that issue equity are more likely to have experienced a history of positive (observable) productivity shocks,

FIGURE C.5: Impulse Responses of Average Price Drop to Alternative Aggregate Shocks



Notes: This figure presents the impulse responses of average price changes during the issuance event to a -1% aggregate TFP shock ($\epsilon^z t = -1\%$), a +25 basis point monetary policy shock ($\epsilon^m t = 0.25\%$), and a +1% lemons shock ($\epsilon_{\eta t} = 1\%$). To incorporate aggregate TFP shocks, we modify the firm's production function to be $y_{jt} = Z_t a_{jt} k_{jt}^{\alpha} l_{jt}^{1-\alpha}$ and assume the aggregate TFP process follows $\log Z_t = 0.95 \log Z_{t-1} + \epsilon_t^z$.

generating the observed runup. Firms who received larger shocks, and therefore have a larger price runup, also do more investment, so the present value of their investment accounts for a larger share of the firm value. Equation (22) shows that the price drop is smaller for such firms.

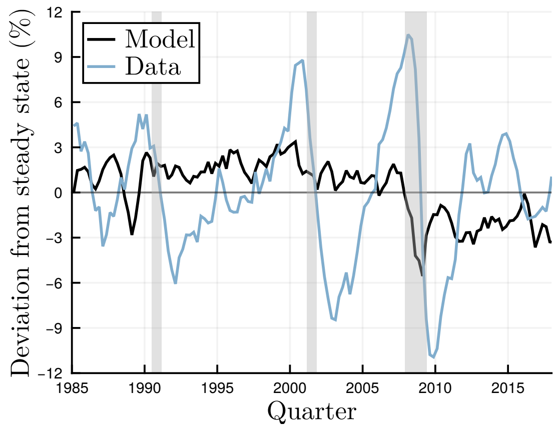
C.3 Additional Results About Lemon Shocks

We now present two additional results on lemons shocks. First, we compare the effects of various aggregate shocks on the aggregate price drop and find that lemons shocks are the only ones capable of matching the data. Second, we quantify the contribution of lemons shocks to aggregate investment fluctuations over the full sample period, not just the GFC.

Response of price drops to other aggregate shocks Figure C.5 plots the impulse response of the average price drops to three contractionary aggregate shock: a lemons shock, a monetary policy shock, and an aggregate TFP shock.[25] Although the monetary shock and TFP shock both lead to a larger average price drop, their effects are an order of magnitude smaller than those of lemons shocks. To put this into context, the standard deviation of the average price drop time-series is approximately 2.2%; generating a price drop response of

[25]For the latter, we incorporate aggregate TFP shocks Z_t into the firm's production function $y_{jt} = Z_t a_{jt} k_{jt}^{\alpha} l_{jt}^{1-\alpha}$ and assume the aggregate TFP process follows $\log Z_t = 0.95, \log Z_{t-1} + \epsilon_t^z$.

Figure C.6: Aggregate Investment Fluctuations Implied by Lemon Shocks vs. Data



Notes: "*Model*" refers to the aggregate investment fluctuations implied solely by the inferred lemons shocks, as shown in Figure 6. "*Data*" refers to the HP-filtered log of real non-residential fixed investment (FRED series `PNFI`). The grey bar is the NBER-dated recession.

this size would require an aggregate TFP shock of -36% or a monetary policy shock of 7,500 basis points. The implausibly large TFP and monetary policy shocks needed to produce a one-standard-deviation change in average price drop implies that observed time-variation in average price drops is primarily driven by lemons shocks. For parsimony, we infer the lemons shock time series assuming they are the only driver of fluctuations in the average price drop.

Contribution of lemons shocks to aggregate investment fluctuations Figure C.6 plots the aggregate investment time series from our model, driven only by the realized lemons shocks, with the empirical investment time series from the data. To focus on business cycle fluctuations, we take logs and apply an HP filter to both the model and data series. Over the observed sample, the model's aggregate investment series has a standard deviation of 1.05%, which is about one-quarter of the observed aggregate investment volatility (4.38%). Moreover, lemons shocks generate significant downturns in aggregate investment during the three recession episodes in our sample, especially during the GFC.

D Extended Model with Information-Insensitive Debt

This appendix provides additional details on the extended model with information-insensitive debt discussed in Section 7.

Table D.1
Extended Model with Debt: Fitted Parameters

Parameter	Description	Value
Debt issuance frictions		
ϕ	Tightness of collateral constraint	0.13
Equity issuance frictions		
σ_η	Capital quality dispersion	0.04
φ_0	Fixed equity issuance costs	0.01
Adjustment Costs		
ψ_0	Scale parameter	1.50
ψ_1	Curvature parameter	0.78
Idiosyncratic productivity shocks		
ρ	Persistence	0.90
σ_a	SD of innovations	0.05
Firm lifecycle		
k_0	New entrants' capital stock	$0.24 \times \mathbb{E}[k]$
a_0	New entrants' idiosyncratic productivity	$0.77 \times \mathbb{E}[a]$

Notes: parameters chosen to match the targets in Table D.2. Labor disutility parameter χ is calibrated to match the steady state level of employment at 1/3. A model period is one quarter.

D.1 Recalibration

The recalibration holds all of the fixed parameters from Table 1 at their same values from the baseline model. We then recalibrate the fitted parameters, contained in Table D.1, to match the targets in Table D.2. As discussed in Section 7, we introduce a new parameter, the collateral constraint tightness ϕ, which we calibrate to match the average net leverage ratio observed in the data. For the remaining parameters, we keep those governing idiosyncratic productivity shocks and adjustment costs at their baseline levels. However, we need to recalibrate the equity issuance frictions to match the share of issuing firms and the average price drop. In addition, because changing financial frictions also affect firm life-cycle dynamics, we adjust the entrant distribution to match the life-cycle moments. Overall, the recalibrated extended model closely matches the same moments as in the baseline model.

D.2 Main Results

Table D.3 shows the aggregate losses from private information in the recalibrated model with debt: the aggregate capital stock is 2% lower and GDP is 1% lower than they would be

Table D.2
Extended Model with Debt: Targeted Moments

Moment	Data	Model
Debt issuance frictions		
Average net leverage ratio	0.10	0.10
Equity issuance frictions		
Average price drop	-3.5%	-3.3%
Frequency of equity issuance (annualized)	18%	18%
Investment frictions		
SD investment rate (annualized)	0.16	0.15
Dividend payout rate (annualized)	4.7%	4.7%
Idiosyncratic productivity shocks		
Autocorrelation of log cash flow rate (annualized)	0.70	0.70
SD of log cash flow rate (annualized)	0.38	0.37
Firm lifecycle		
Young vs. old log capital gap	-1.67	-1.67
Young vs. old growth rate gap	16.7pp	16.7pp

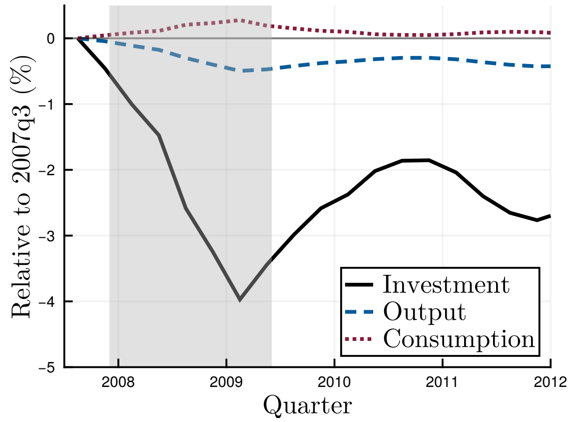
Notes: moments targeted to pin down the parameters in Table D.1. The net leverage ratio is measured as the ratio between net debt, which is the total of current liabilities (`DLCQ`) and long-term debt (`DLTTQ`) net of cash and short-term investments (`CHEQ`), and total book value assets (`ATQ`). The average price drop is the average price change described in the main text, weighted by new shares issued (in order to not over-emphasize small events). The frequency of equity issuance is the average of the fraction of firms with positive total equity issuance, after applying the filter proposed by McKeon (2015), in each year. The investment rate is computed as the ratio of capital expenditures (`CAPXQ`) to the lagged total book value assets (`ATQ`), expressed as an annual rate. The dividend payout rate is aggregated payout, which equals to the sum of cash dividends (`DVY`) and purchase of common and preferred stock (`PRSTKCY`), relative to lagged total book value assets, also expressed as an annual rate. The firm lifecycle targets the estimated $\hat{\beta}_{old}$ in regression (23), where y_{jt} is either log capital or the log capital growth rate.

Table D.3
Extended Model with Debt: Steady State Losses from Private Information

Capital stock	Employment	Wages	Output
-2.1%	-0.5%	-0.5%	-1.0%

Notes: steady state macro aggregates relative to full information benchmark from Section 3.

Figure D.1: Extended Model with Debt: Role of Lemon Shocks during Great Financial Crisis



Notes: This figure plots the time series produced by the extended model with debt in response to the realized sequence of lemons shocks $\sigma_{\eta t}$ we infer from (26) between 1985q1-2018q4, focusing on the 2007q3-2012q1 window. The y-axis is the percentage deviation from 2007q3 and the grey bar is the NBER-dated recession.

under full information. While these losses are large, they are about half as big as the model without debt. The reason is, as described in the main text, the recalibrated dispersion of capital quality is lower in the model with debt.

Figure D.1 plots the effects of the lemons shock during the GFC in the model with debt. We infer the realized shocks from the data using the same methodology from Section 6 in this extended model. The lemons shocks imply a 4% decline in aggregate investment, explaining about a quarter of the observed decline in investment over this period.